Pinning Controllability Analysis of Complex Networks With a Distributed Event-Triggered Mechanism

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***Abstract*—Pinning control synchronization of complex networks is a fascinating and hot issue in the field of nonlinear science. However, the existing works are all based on a continuous-time feedback control strategy and assume that each network node can have continuous access to the states of its neighbors. This brief presents a novel distributed event-triggered mechanism for pinning control synchronization of complex networks. The control of nodes is only triggered at their own event time, which effectively reduces the frequency of controller updates compared with continuous-time feedback control. Considering limited communication, the new approach successfully avoids the continuous communication used for calculating the error thresholds in the event-triggered mechanism. In addition, we also develop a new alternative iterative algorithm that can further reduce the consumption of computing and communication resources to some extent. Finally, simulation results show the effectiveness of the proposed approaches and illustrate the correctness of the theoretical results.**

***Index Terms*—Complex networks, distributed event triggering, iterative algorithm, pinning controllability.**

I. Introduction

COMPLEX networks that consist of vast interconnected nodes are ubiquitous in the real world. What we often encounter, such as social networks, biological networks, electrical power grids, the Internet, and World Wide Web, can all be modeled and analyzed by complex networks [1], [2]. During the past decade, synchronization control has attracted increasing attention from various fields of science and engineering as a main issue of complex networks [3]–[5].

For large-scale networks, we often adopt the pinning control strategy to drive the network from any initial state to a desired synchronous state by applying local control actions to a small fraction of network nodes, which avoids implementing control input to all network nodes. In such a case the network is called *globally pinning controllable*, which has been intensively investigated in [6]–[10]. However, the previous works are all based on continuous-time feedback control techniques that have an obvious insurmountable deficiency: the designed control law requires real-time updates, which promotes network nodes to be equipped with high-performance processors and high-speed communication channels. More rigorously, it is assumed that each node can have continuous access to the states of its neighbors. In many actual applications the nodes in a network transmit their own state values such as position, velocity, and heading to their individual neighbors not continuously but at some discrete time instants. Therefore, we urgently need to develop a new approach to overcome these problems while preserving the nice properties of stability and convergence.

An event-triggered control mechanism offers a new point of view on how information could be sampled for control purposes. In a network, an autonomous node transmits its local state to its neighbors only when it is necessary; that is, only when a measurement of the local node state error reaches a specified threshold [11], [12]. Tabuada [11] seminally presented a triggering condition based on norms of the state and the state error $e = x(t_k) - x(t)$; that is, the last measured state minus the current state of this node, in which the measurement received at the controller is held a constant until a new measurement arrives. When this happens the error is set to zero and starts increasing until it triggers a new measurement update. The last years also witnessed the event-triggered control mechanism's better robust and higher efficient usage of network bandwidth in networked control systems [13]–[15].

The work in this brief is to introduce this event-triggered mechanism into pinning control synchronization of directed complex networks with nonlinear dynamics. Note that this introduction is based on an improved and better event-triggered mechanism, which further reduces actuation update times and communication traffic compared with [11] and [12]. Through this technology introduction we solve the continuous-time feedback problem very well and achieve the purpose of communication reduction in pinning control synchronization of complex networks. Furthermore, based on the idea presented in [16], we also develop an alternative distributed iterative algorithm to determine each node's event-triggering time in advance, which will consume less costly computing and communication resources to some extent. The main contribution of this brief is that it focuses on introducing the event-triggered mechanism of multiagent systems into the study of pinning control synchronization in complex networks and proposes some further improvements that can enhance the performance of the mechanism in actual implementation.

The remainder of this brief is organized as follows. Section II presents the new model, stability criteria, and the details of the proof of main results. Section III develops an alternative distributed iterative algorithm for the event-triggered mechanism. Section IV gives some numerical simulations to verify the main results. Finally, this brief is concluded in Section V.

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II. Basic Stability Criteria for Pinning Controllability Synchronization Based on Distributed Event-Triggered Mechanism

Consider a linearly coupled complex network composed of N identical nodes, in which each node is an n-dimensional dynamical system described by

$$\dot{x}_i(t)=f\left(x_i(t),t\right)+c\sum_{j\in\mathcal{N}_i} w_{ij}\Gamma\left(x_j(t)-x_i(t)\right),\ i=1,\dots,N \tag{1}$$

where $x_i(t)\in R^n$ is the state of node i, $f(x_i(t),t)\in R^n$ is a continuous vector-valued function, $\mathcal{N}_i$ is the neighbor set of node i, the positive constant c represents the coupling strength, the inner coupling matrix $\Gamma\in R^{n\times n}$ is positive definite and describes an inner coupling between the subsystems, and the matrix $\mathcal{W}=(w_{ij})\in R^{N\times N}$ represents the coupling configuration defined as follows: w_{ij} is a positive weight value if there is a directed edge from node j to node i, and $w_{ij}=0$, otherwise.

The isolated node of network (1) is given by

$$\dot{s}(t)=f\left(s(t),t\right) \tag{2}$$

where $s(t)=(s_1(t),\dots,s_n(t))^T\in R^n$. Note that we should pin all roots of directed trees in a directed network at least such that the isolated node and all the network nodes can contain a spanning directed tree. If a directed tree can have several roots, then we select the root with the highest out-weight sum preferentially, because it has the most effect on other nodes.

Our control goal is to synchronize complex network (1) to the homogenous trajectory (2). The previous most common models are investigated in [6]–[10], [18], and [19]; however, their insurmountable deficiency is very obvious: each node's controller implementation must realize its all neighbors' current states at every time instant, which means that continuous information communication between autonomous nodes is needed, promoting for nodes to be equipped with high-performance processors and communication facilities. Therefore, these traditional pinning control strategies undoubtedly will consume much more energy and be largely restricted in practical applications with limited computing resources and network bandwidth.

To introduce the event-triggered mechanism, assume that the triggering time sequence of node i is $0=t_0^i,t_1^i,\dots,t_{k_i}^i$ and that each node can only have access to its neighbors' states. At time instant $t_{k_i}^i$, node i and its neighbor nodes measure their own states, respectively denoted by $x_i(t_{k_i}^i)$ and $x_j(t_{k_i}^i),\ j\in\mathcal{N}_i$, and take the measurements as their target points, which remain unchanged until the next triggering time instant $t_{k_i+1}^i$ comes. Therefore, a completely new pinning control model based on the distributed event-triggered mechanism is proposed as

$$\dot{x}_i(t)=f\left(x_i(t),t\right)+u_i(t),\quad i=1,\dots,N \tag{3}$$

where the controller of node i is defined in detail as

$$\begin{aligned}u_i(t)=&\,c\sum_{j\in\mathcal{N}_i} w_{ij}\Gamma\left(x_j\left(t_{k_i}^i\right)-x_i\left(t_{k_i}^i\right)\right)\\&+cw_{i0}\Gamma\left(s\left(t_{k_i}^i\right)-x_i\left(t_{k_i}^i\right)\right),\quad t\in\left[t_{k_i}^i,t_{k_i+1}^i\right)\end{aligned} \tag{4}$$

where the pinning control feedback gains are defined as follows: If $i\in\mathcal{V}_{\mathrm{pin}}$, $w_{i0}>0$; otherwise, $w_{i0}=0$. Here, $\mathcal{V}_{\mathrm{pin}}=\{i_1,\dots,i_l\}$ denotes the set of the pinned nodes. Let $\mathcal{W}_0=\mathrm{diag}\{w_{10},w_{20},\dots,w_{N0}\}$.

Note that this new event-triggered mechanism is totally different from the existing schemes in [11]–[15], in which each node i executes triggering only at its own individual event-triggering time instant t_k^i and then updates its controller by utilizing the measurement states of node i, neighbors of node i, and isolated node s. That is to say, in time interval $[t_{k_i}^i,t_{k_i+1}^i)$, the controller of each node i will remain unchanged as a constant $u_i(t_{k_i}^i)$ until its next triggering time instant $t_{k_i+1}^i$ comes. Therefore, the number of control updates can be largely reduced as well as the communication between different nodes in the network.

In the following, we begin by presenting some stability criteria to guarantee feasibility of the new model. First, we need to define the following three types of measurement errors in time interval $[t_{k_i}^i,t_{k_i+1}^i)$ as $e_i(t)=x_i(t_{k_i}^i)-x_i(t)$, $e_{ij}(t)=x_j(t_{k_i}^i)-x_j(t),\ j\in\mathcal{N}_i$; $e_{is}(t)=s(t_{k_i}^i)-s(t),\ i\in\mathcal{V}_{\mathrm{pin}}$. The measurement error represents the degree that the present time states deviates from the last sample time states, and when the measurement error of node i reaches a time-varying threshold prescribed in advance, the event is triggered, and then, node i begins to update its controller.

Let $\varepsilon_i(t)=x_i(t)-s(t),\ \ i=1,\dots,N$, from (2)–(4), we can have the following error system:

$$\begin{aligned}\dot{\varepsilon}_i(t)=&\,f\left(x_i(t),t\right)-f\left(s(t),t\right)+c\sum_{j\in\mathcal{N}_i} w_{ij}\Gamma e_{ij}(t)\\&-cd_i\Gamma e_i(t)+c\sum_{j\in\mathcal{N}_i} w_{ij}\Gamma\varepsilon_j(t)-cd_i\Gamma\varepsilon_i(t)\\&+cw_{i0}\Gamma e_{is}(t)-cw_{i0}\Gamma e_i(t)-cw_{i0}\Gamma\varepsilon_i(t)\end{aligned} \tag{5}$$

where $d_i=\sum_{j\in\mathcal{N}_i} w_{ij},\ i,j\in\mathcal{V}$.

Let $F(x(t),t)-F(s(t),t)=(f^T(x_1(t),t)-f^T(s(t),t),\dots,f^T(x_N(t),t)-f^T(s(t),t))^T$, $\varepsilon(t)=(\varepsilon_1^T(t),\dots,\varepsilon_N^T(t))^T$, $e(t)=(e_1^T(t),\dots,e_N^T(t))^T$, $\tilde{e}_i(t)=(e_{i1}^T(t),\dots,e_{iN}^T(t))^T$, $\tilde{e}(t)=(\tilde{e}_1^T(t),\dots,\tilde{e}_N^T(t))^T$, $e_s(t)=(e_{1s}^T(t),\dots,e_{Ns}^T(t))^T$. Then, error system (5) can be rewritten in compact matrix form as

$$\begin{aligned}\dot{\varepsilon}(t)=&\,F\left(x(t),t\right)-F\left(s(t),t\right)+(M_1\otimes\Gamma)\varepsilon(t)+(c\mathcal{W}\otimes\Gamma)\varepsilon(t)\\&+(M_1\otimes\Gamma)e(t)+(c\hat{W}\otimes\Gamma)\tilde{e}(t)+(M_2\otimes\Gamma)e_s(t)\end{aligned} \tag{6}$$

where $M_1=-cD-M_2$, $D=\mathrm{diag}\{d_1,\dots,d_N\}$, $M_2=\mathrm{diag}\{cw_{10},\dots,cw_{N0}\}$, $\hat{W}=\mathrm{diag}\{W_1,\dots,W_N\}\in R^{N\times N^2}$ with $W_i=(w_{i1},\dots,w_{iN})\in R^{1\times N}$.

Consider that each node can only obtain its neighbors' measurements, the event is also computed only depending on local information that is available to each node. We propose the following event-triggering threshold:

$$\|e_i(t)\|+\|\tilde{e}_i(t)\|+\|e_{is}(t)\|=\beta\sqrt{\sum_{j\in\mathcal{N}_i}\left\|x_j\left(t_{k_i}^i\right)-x_i\left(t_{k_i}^i\right)\right\|^2} \tag{7}$$

where $\beta>0$.

When an event is triggered by node i, we have $e_i(t_{k_i}^i)=e_{ij}(t_{k_i}^i)=e_{is}(t_{k_i}^i)=0$ because $t=t_{k_i}^i$ is an event time for node i. Note that the triggering condition (7) can guarantee that

$$\|e_i(t)\|+\|\tilde{e}_i(t)\|+\|e_{is}(t)\| \le \beta\sqrt{\sum_{j\in\mathcal{N}_i}\left\|x_j\left(t^i_{k_i}\right)-x_i\left(t^i_{k_i}\right)\right\|^2} \tag{8}$$

is satisfied. The next result reveals the convergence for pinning synchronization using threshold (7).

Assumption 1—[20]: For the nonlinear function f in the network (3), there exists two positive constants θ and K such that for $\forall\, x, y \in R^n$ and $t > 0$, we have

$$\begin{aligned}(y-x)^T\left(f(y,t)-f(x,t)\right) &\le \theta(y-x)^T\Gamma(y-x)\\ \|f\left(x(t),t\right)-f\left(y(t),t\right)\| &\le K\|x(t)-y(t)\|.\end{aligned} \tag{9}$$

Note that functions satisfying the second inequality of (9) are called Lipschitz functions, whereas functions satisfying the first inequality are called one-sided Lipschitz functions. It can be immediately proved that a Lipschitz function with Lipschitz constant K is also one-sided Lipschitz with the same constant.

Theorem 1: Suppose that the underlying directed graph consisting of pinning control network (3) and isolated node (2) has a directed spanning tree, which takes the isolated node (2) as a root. Then, globally asymptotical synchronization in network (3) can be reached if

$$\min_{i\in\mathcal{V}}\mathcal{R}_e\left(\lambda_i(L+\mathcal{W}_0)\right) > (\theta+\kappa\mu)/c \tag{10}$$

where L is the Laplacian matrix of network (3), and κ, μ are estimated in (11) and (14).

Proof: We first have the following derivation:

$$\begin{aligned}&\left(\|e_i(t)\|+\|\tilde{e}_i(t)\|+\|e_{is}(t)\|\right)^2\\ &\le \beta^2\sum_{j\in\mathcal{N}_i}\left\|x_j\left(t^i_{k_i}\right)-x_i\left(t^i_{k_i}\right)\right\|^2\\ &= \beta^2\sum_{j\in\mathcal{N}_i}\|x_j(t)+e_{ij}(t)-x_i(t)-e_i(t)\|^2\\ &\le 2\beta^2\sum_{j\in\mathcal{N}_i}\left(\|\varepsilon_j(t)-\varepsilon_i(t)\|^2+\|e_{ij}(t)-e_i(t)\|^2\right)\\ &\le 2\beta^2\left[2N\|\varepsilon(t)\|^2+N\left(\|e_i(t)\|+\|\tilde{e}_i(t)\|+\|e_{is}(t)\|\right)^2\right].\end{aligned}$$

Then, we can have $\|e_i(t)\|+\|\tilde{e}_i(t)\|+\|e_{is}(t)\| \le l_i\|\varepsilon(t)\|$, $l_i = 2\beta\sqrt{N/(1-2N\beta^2)}$. Thus, we can further obtain

$$\begin{aligned}\|M_1\otimes I_n\|\,\|e_i(t)\| &\le l_i\|M_1\otimes I_n\|\,\|\varepsilon(t)\|\\ \|c\hat{W}\otimes I_n\|\,\|\tilde{e}_i(t)\| &\le l_i\|c\hat{W}\otimes I_n\|\,\|\varepsilon(t)\|\\ \|M_2\otimes I_n\|\,\|e_{is}(t)\| &\le l_i\|M_2\otimes I_n\|\,\|\varepsilon(t)\|.\end{aligned}$$

Finally, we can get

$$\begin{aligned}&\|M_1\otimes I_n\|\,\|e(t)\|+\|c\hat{W}\otimes I_n\|\,\|\tilde{e}(t)\|\\ &\quad+\|M_2\otimes I_n\|\,\|e_s(t)\| \le \kappa\|\varepsilon(t)\|\end{aligned} \tag{11}$$

where $\kappa = 3l_i\eta\sqrt{N}$, $\eta = \max\{\|M_1\|, \|c\hat{W}\|, \|M_2\|\}$.

Let λ_i, $i\in\mathcal{V}$ be an eigenvalue of matrix $L+\mathcal{W}_0$. It is evident that $-c\lambda_i+(\theta+\kappa\mu)$ is an eigenvalue of matrix $-c(L+\mathcal{W}_0)+(\theta+\kappa\mu)I_N$. Under pinning control condition (10), we know that $\mathcal{R}_e(c\lambda_i-(\theta+\kappa\mu)) > 0$ holds for all $i\in\mathcal{V}$. Then, we can conclude that $c(L+\mathcal{W}_0)-(\theta+\kappa\mu)I_N$ is an M-matrix [17] and that there exists a positive definite diagonal matrix $\Xi = \mathrm{diag}\{\xi_1,\ldots,\xi_N\} > 0$ such that

$$\left[\Xi\left(c(L+\mathcal{W}_0)-(\theta+\kappa\mu)I_N\right)\right]_s > 0. \tag{12}$$

Construct the following Lyapunov functional candidate:

$$V(t) = \frac{1}{2}\varepsilon^T(t)(\Xi\otimes I_n)\varepsilon(t). \tag{13}$$

Combine Assumption 1 and inequality (11), we have

$$\begin{aligned}\dot{V}(t) &= \varepsilon^T(t)(\Xi\otimes I_n)\\ &\quad\times\Big[F\left(x(t),t\right)-F\left(s(t),t\right)+(M_1\otimes\Gamma)\varepsilon(t)\\ &\qquad+(c\mathcal{W}\otimes\Gamma)\varepsilon(t)+(M_1\otimes\Gamma)e(t)\\ &\qquad+(c\hat{W}\otimes\Gamma)\tilde{e}(t)+(M_2\otimes\Gamma)e_s(t)\Big]\\ &\le -c\varepsilon^T(t)\left[\Xi(L+\mathcal{W}_0)\otimes\Gamma\right]\varepsilon(t)\\ &\quad+\theta\varepsilon^T(t)(\Xi\otimes\Gamma)\varepsilon(t)+\kappa\|\Xi\otimes\Gamma\|\varepsilon^T(t)\varepsilon(t)\\ &\le -c\varepsilon^T(t)\left[\Xi(L+\mathcal{W}_0)\otimes\Gamma\right]\varepsilon(t)+\theta\varepsilon^T(t)(\Xi\otimes\Gamma)\varepsilon(t)\\ &\quad+\kappa\frac{\|\Xi\otimes\Gamma\|}{\lambda_{\min}(\Xi\otimes\Gamma)}\varepsilon^T(t)(\Xi\otimes\Gamma)\varepsilon(t)\\ &= -\varepsilon^T(t)\left[\Xi\left(c(L+\mathcal{W}_0)-(\theta+\kappa\mu)I_N\right)\right]_s\otimes\Gamma\varepsilon(t)\end{aligned} \tag{14}$$

where $\mu = \|\Xi\otimes\Gamma\|/\lambda_{\min}(\Xi\otimes\Gamma)$.

Considering $\Gamma > 0$ and inequality (12), we have $\dot{V}(t) < 0$ for all $\varepsilon \ne 0_{Nn}$. Therefore, the error system (6) is globally asymptotically stable at the origin. Then, it follows that the pinning control network (3) globally asymptotically synchronizes to the isolated node (2).

III. Alternative Distributed Iterative Event-Triggered Algorithm

It is evident that the given approach exits a serious deficiency that each node has to judge whether the individual event is triggered or not, which requires calculating the individual three errors and threshold utilizing local nodes' state information at every time. Here, we will propose a distributed iterative algorithm to overcome this problem.

For node i, let $t^{i*}_{k_i+1}$ be the next triggering time instant determined in the given approach. Since the continuity of $e_i(t)+\tilde{e}_i(t)+e_{is}(t)$ and $\sqrt{\sum_{j\in\mathcal{N}_i}\|x_j(t^i_{k_i})-x_i(t^i_{k_i})\|^2}$, $t^{i*}_{k_i+1}$ is the first time instant after $t^i_{k_i}$ such that $\|e_i(t)\|+\|\tilde{e}_i(t)\|+\|e_{is}(t)\| = \beta\sqrt{\sum_{j\in\mathcal{N}_i}\|x_j(t^i_{k_i})-x_i(t^i_{k_i})\|^2}$. Thus, for any $\tau\in[t^i_{k_i}, t^{i*}_{k_i+1})$, the upper bound of $\|e_i(t)\|+\|\tilde{e}_i(t)\|+\|e_{is}(t)\|$ is not larger than $\beta\sqrt{\sum_{j\in\mathcal{N}_i}\|x_j(t^i_{k_i})-x_i(t^i_{k_i})\|^2}$, which implies that (8) holds, and thus, $\dot{V}(t) \le 0$ in time interval $[t^i_{k_i}, \tau)$. This means that τ is a valid choice for the next triggering time instant $t^i_{k_i+1}$.

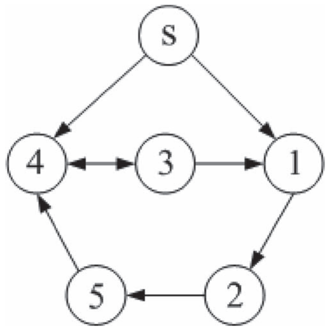

Fig. 1. Interaction diagraph of five nodes, and node s is the isolated node.

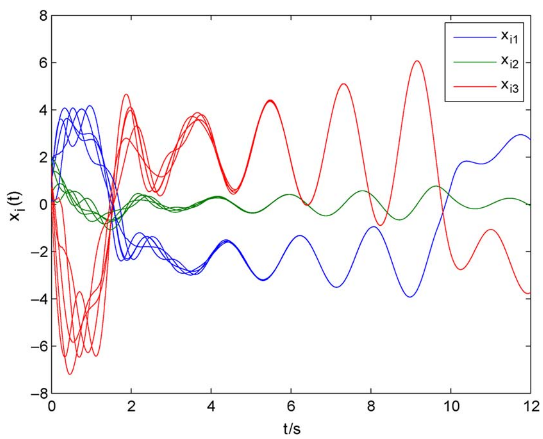

Fig. 2. Evolution of states $x_i(t)$, $i = 1, 2, \ldots, 5$.

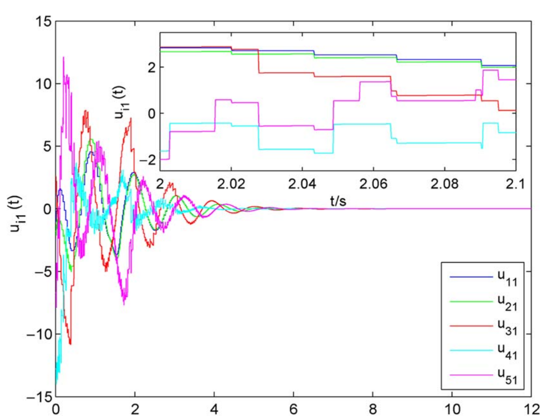

Fig. 3. Evolution of control signals $u_{i1}(t)$, $i = 1, 2, \ldots, 5$.

Let $E_i = e_i(t) + \tilde{e}_i(t) + e_{is}(t)$, we have

$$\begin{aligned}\frac{d}{dt}\|E_i\| &\le \left\|\dot{e}_i(t)+\dot{\tilde{e}}_i(t)+\dot{e}_{is}(t)\right\| \\ &\le \left\|f\left(x_i(t),t\right)+u_i\left(t^i_{k_i(t)}\right)\right\|+\left\|\dot{X}_i(t)\right\|+\|f(s(t),t)\| \\ &\le K\left\|x_i(t)\right\|+\left\|u_i\left(t^i_{k_i(t)}\right)\right\|+\left\|\tilde{X}_i(t)\right\|+K\left\|s(t)\right\| \quad (15)\end{aligned}$$

where $k_i(t) = \arg\max_{k\in N^+}\{t^i_k | t^i_k \le t\}$, K is a Lipschitz constant; $\dot{X}_i(t) = (\dot{x}^T_{j_1}(t), \dot{x}^T_{j_2}(t), \ldots, \dot{x}^T_{j_N}(t))^T$, if $j_p \in \mathcal{N}_i$, then $\dot{x}_{j_p} = f(x_{j_p}(t), t) + u_{j_p}(t^{j_p}_{k_{j_p}(t)})$, $p = 1, 2, \ldots, N$; otherwise, $\dot{x}_{j_p} = 0$; $\tilde{X}_i(t) = (S^2_{j_1}, S^2_{j_1}, \ldots, S^2_{j_N})^T$, if $j_p \in \mathcal{N}_i$, then $S_{j_p} = K\|x_{j_p}(t)\| + \|u_{j_p}(t^{j_p}_{k_{j_p}(t)})\|$, $p = 1, 2, \ldots, N$; otherwise, $S_{j_p} = 0$. Furthermore, we let

$$s_0 = \beta\sqrt{\sum_{j\in\mathcal{N}_i}\left\|x_j\left(t^i_{k_i}\right) - x_i\left(t^i_{k_i}\right)\right\|^2} \quad (16)$$

$$\eta_i(t) = K\left\|x_i(t)\right\| + \left\|u_i\left(t^i_{k_i(t)}\right)\right\| + \left\|\tilde{X}_i(t)\right\| + K\left\|s(t)\right\|. \quad (17)$$

In time interval $[t^i_{k_i}, t^{i*}_{k_i+1})$, let Δ be the accurate time consumption that $\|e_i(t)\| + \|\tilde{e}_i(t)\| + \|e_{is}(t)\|$ increases from 0 to s_0. From (15), we can know that the increasing rate of $\|e_i(t) + \tilde{e}_i(t) + e_{is}(t)\|$ is lower than $\eta_0 = \eta_i(t^i_{k_i})$. Then, $\tau_0 = t^i_{k_i} + s_0/\eta_0 < t^i_{k_i} + \Delta = t^{i*}_{k_i+1}$; thus, τ_0 will be a suitable conservative candidate for $t^i_{k_i+1}$ and $\dot{V}(t) \le 0$ in $[t^i_{k_i}, \tau_0)$. Therefore, for node i, the next triggering time instant can be given by

$$t^i_{k_i+1} = t^i_{k_i} + s_0/\eta_0 \quad (18)$$

where $\eta_0 = \eta_i(t^i_{k_i})$. Note that the determination of node i's next triggering time instant τ_0 is only dependent on the current time's state values of node i and isolated node s as well as the state measurement values received from node i's neighbors at time instant $t^i_{k_i}$, which avoids the judgement as to whether each node's event happens or not.

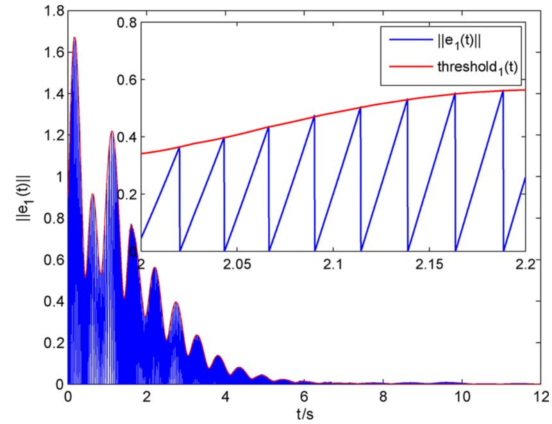

Fig. 4. Evolution of measurement error and threshold.

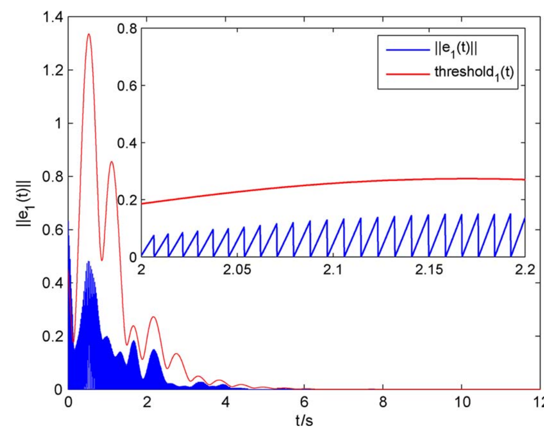

Fig. 5. Evolution of measurement error and threshold in iterative algorithm.

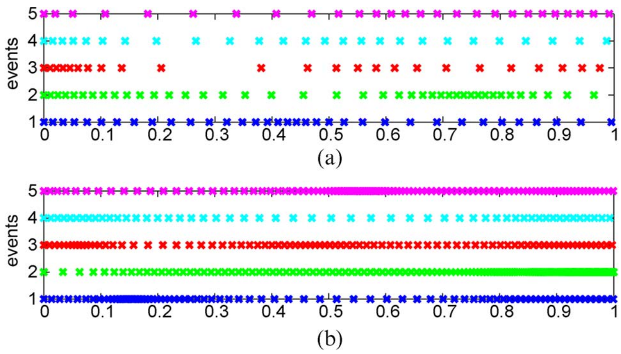

Fig. 6. Event-triggering times in different methods. (a) The events of nodes in the basic event-triggered approach. (b) The events of nodes in the iterative algorithm.

IV. Simulation Results

Here, we will provide some simulations to illustrate the proposed approaches. Consider the information interactive network with communication graph G given in Fig. 1, and the corresponding Laplacian matrix is given by

$$L = \begin{bmatrix} 0.31 & 0 & -0.31 & 0 & 0 \\ -0.37 & 0.37 & 0 & 0 & 0 \\ 0 & 0 & 0.42 & -0.42 & 0 \\ 0 & 0 & -0.42 & 0.69 & -0.27 \\ 0 & -0.58 & 0 & 0 & 0.58 \end{bmatrix}.$$

Each network node's kinematic under study is modeled by a chaotic Chua's circuit, which is described by $\dot{x}_i(t) = f(x_i(t), t) = (10(x_{i2} - g(x_{i1})), x_{i1} - x_{i2} + x_{i3}, -18x_{i2})^T$, where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, and $g(x_{i1}) = 1/4x_{i1} + 1/2(-1/3 - 1/4)(|x_{i1} + 1| - |x_{i1} - 1|)$. We have assumed that the first and the fourth network node are the pinned nodes and set $c = 5$, $\mathcal{W}_0 = \mathrm{diag}\{0.25, 0, 0, 0.45, 0\}$, $\Gamma = \mathrm{diag}\{2, 1, 1\}$, $\beta = 0.5$, $K = 3$. The initial values of network nodes and the virtual leader node are randomly generated in the interval [0, 2].

The simulation results are shown in Figs. 2–6. Fig. 2 shows the evolution of all node states $x_i(t)$. Fig. 3 shows the piecewise constant control signals $u_{i1}(t)$. The evolution of measurement errors is shown in Fig. 4. Fig. 5 shows the evolution of measurement errors under the iterative algorithm, since the next triggering time is selected smaller than t_k^i determined by (7), the error norm will always be strictly smaller than $\beta\sqrt{\sum_{j\in\mathcal{N}_i} \|x_j(t_{k_i}^i) - x_i(t_{k_i}^i)\|^2}$. In Fig. 6, the events of each node are marked under the basic event-triggered approach and the iterative algorithm in the time interval [0,1], respectively, from which we can see that the sampling is sporadic rather than every time instant.

Comparing plot (a) with plot (b) in Fig. 6, we can find that event times in the latter is more than that in the former. Then we can obtain this as fact: although the iterative algorithm avoids the judgement as to whether each node's event happens or not, it also add the event-triggering times in the network nodes' synchronization process compared with the basic event-triggered approach.

V. Conclusion

In this brief we have introduced a novel distributed event-triggered mechanism into pinning control synchronization of complex networks. We proposed a basic event-triggered scheme and presented the convergence analysis. In addition, we also developed a new alternative distributed iterative algorithm that further reduces the consumption of computing and communication resources. Future work will include extending the proposed approach to complex networks with communication delays and disturbances.

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