Robust Finite-Time Dynamic Average Consensus With Exponential Convergence Rates

Kedong Xu, Lan Gao, Fei Chen, *Senior Member, IEEE*, Chaojie Li, *Member, IEEE*, and Qi Xuan

***Abstract*—This brief focuses on the dynamic weighted average consensus problem and aims to achieve accurate tracking of the weighted average of all the time-varying reference signals in a network. We first propose a robust dynamic weighted average consensus (RDWAC) algorithm that employs a simple fixed control gain and introduces an individual weight for each agent compared with recent works. Furthermore, a theoretical finite-time convergence analysis instead of an asymptotic one is provided by constructing a novel Lyapunov function, which shows that the accurate weighted average consensus can be reached exponentially within a finite time interval. In addition, the lower bound of the required convergence time is given and the relationship between the lower bound and the initial steady-state error and control parameters is established explicitly. Finally, some numerical examples are given to illustrate the effectiveness of the proposed algorithm.**

***Index Terms*—Dynamic average consensus, weighted average consensus, finite-time convergence, multi-agent networks.**

I. Introduction

Given a connected network with a group of agents each of which has its own local time-varying reference signal, dynamic average consensus (DAC) aims to design a distributed algorithm to ensure that each agent can track the average of all the time-varying reference signals by exchanging local information with its neighbors [1]. During the past decade, there has been more and more attention devoted to dynamic average consensus due to its significance in various applications such as formation control [2], sensor fusion [3], robotic manipulator control [4], [5] and distributed optimization [6], [7].

The dynamic average consensus problem is first investigated in [1] and the authors present a distributed algorithm to achieve average consensus on the time-varying input signals with steady-state values. Though a standard frequency-domain technique is employed for the convergence analysis, the DAC algorithm in [1] depends on the derivatives of reference signals explicitly, which leads to certain limitations in practical applications. To circumvent this issue, the authors in [8] propose two novel dynamic consensus algorithms: proportional (P) algorithm and proportional-integral (PI) algorithm. However, the P algorithm can not guarantee zero steady-state error with arbitrary estimator initializations and the PI algorithm can only eliminate the steady-state error in the case of constant inputs. The PI algorithm is further extended in [9] to achieve zero steady-state error for a certain class of reference signals. It is worth pointing out that the aforementioned algorithms are almost based on continuous functions and also suffer from the nonzero steady-state error. In contrast, a discontinuous signum function is introduced into the DAC algorithm in [10], which provides a new viewpoint on the design of DAC algorithms. The discontinuous algorithm is extended to the networked Euler-Lagrange systems [11], [12] and the reference signals with bounded accelerations [13]. Although the proposed algorithm in [10] is able to track the accurate average of the time-varying reference signals, a specific initialization on the internal states has to be guaranteed, which implies that the algorithm requires a reinitialization once some agents leave or join the communication network. Also, the event-triggered control strategies in the leaderless or leader-following consensus are studied in [14], [15].

More recently, a robust DAC algorithm is proposed in [16], [17], where both the zero steady-state error and the robustness of initialization can be guaranteed at the cost of two rounds of communication between agents. To further reduce the communication cost, an improved robust DAC algorithm with a built-in singularly perturbed system is also proposed in [16]. On the other hand, two alternative DAC algorithms with one round of communication for undirected and directed networks are proposed in [18], [19], in which the authors show that not only zero or arbitrarily small steady-state error can be achieved with initialization robustness but the full knowledge and time derivatives of the dynamics generating reference signals are not required. However, there still exist several issues that need to be solved in the recent works [18], [19]. First, the proposed adaptive control gain not only needs to be computed

Manuscript received January 13, 2021; accepted January 18, 2021. Date of publication January 25, 2021; date of current version June 29, 2021. This work was supported in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LQ20F030020; in part by the National Natural Science Foundation of China under Grant 61973061; in part by the Hebei Natural Science Foundation for Distinguished Young Scholars under Grant F2019501043; and in part by the Australian Research Council under Grant DP200101197 and Grant DE210100274. This brief was recommended by Associate Editor S. Li. *(Corresponding author: Lan Gao.)*

Kedong Xu and Qi Xuan are with the Institute of Cyberspace Security, School of Information Engineering, Zhejiang University of Technology, Hangzhou 310023, China (e-mail: xukedong96@163.com; qixuan@zjut.edu.cn).

Lan Gao is with Hangzhou Innovation Institute, Beihang University, Hangzhou 310051, China (e-mail: langaouc@gmail.com).

Fei Chen is with the School of Control Engineering, Northeastern University at Qinhuangdao, Qinhuangdao 066004, China (e-mail: fei.chen@ieee.org).

Chaojie Li is with the School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, NSW 2052, Australia (e-mail: cjlee.cqu@163.com).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCSII.2021.3054039.

Digital Object Identifier 10.1109/TCSII.2021.3054039

instantly at all time but the amount of control gains in each node might be multiple since it depends on the amount of neighbors of each node. Second, the given stability analysis for the case of undirected networks is based on the asymptotic convergence theorem, which implies that the accurate dynamic consensus can be achieved asymptotically only when the time horizon tends to infinity. Third, the tracking objective of DAC algorithms studied in [18], [19] mainly considers a simple or special case (absolute signal average), which might not satisfy the customized requirement in practical applications.

In this brief, we first propose a robust dynamic weighted average consensus (RDWAC) algorithm that aims to track a customized objective signal instead of an absolute signal average. The proposed RDWAC algorithm employs a simple fixed control gain and introduces an individual weight for each agent compared with the recent works [18], [19], which ensures that each agent is able to track the weighted average of all the time-varying reference signals. Furthermore, a novel quadratic Lyapunov function is constructed to complete the finite-time convergence analysis instead of an asymptotic one. The convergence analysis shows that the accurate weighted average of time-varying reference signals can be achieved exponentially within a finite time interval when the control parameters satisfy certain conditions. In addition, the lower bound of the required convergence time is given and the relationship between the lower bound and the initial steady-state error and control parameters is established explicitly. Finally, some numerical examples are provided for comparison to validate the proposed RDWAC algorithm. It is worthwhile to mention that these results extend the robust DAC results in [18], [19] and the proposed RDWAC algorithm also inherits the good performance, i.e., the zero steady-state error and the initialization robustness can be both guaranteed.

II. Preliminaries and Problem Formulation

A. Graph Theory

A connected undirected graph composed of N nodes is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{v_1, \dots, v_N\}$ denotes the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edge set, and $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ denotes the adjacency matrix where $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The neighbor set of node i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V}, (v_i, v_j) \in \mathcal{E}\}$ and the number of neighbors of node i is denoted as $N_i = |\mathcal{N}_i|$. The Laplacian matrix associated with the adjacency matrix A is defined by $L = (l_{ij}) \in \mathbb{R}^{N\times N}$, where $l_{ij} = -a_{ij}, i \neq j$ and $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$. Here L is a symmetric positive semidefinite matrix and its eigenvalues are denoted as $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ in an ascending order. The incidence matrix of graph $\mathcal{G}$ is defined by $B = (b_{ij}) \in \{-1, 0, 1\}^{N\times \ell}$, where $b_{ij} = -1$ if edge e_j leaves node v_i, $b_{ij} = 1$ if edge e_j enters node v_i, and $b_{ij} = 0$ otherwise. Note that each undirected edge can be considered as two distinct directed edges.

B. Dynamic Average Consensus (DAC)

Consider a multi-agent network composed of N agents where each agent i contains a time-varying reference signal $\phi_i(t) \in \mathbb{R}^N$. The dynamic average consensus problem studies how to design a distributed algorithm to ensure that each agent i can track the average value of all the time-varying reference signals. The tracking objective is given by

$$\bar{\phi}(t) = \frac{1}{N}\sum_{i=1}^{N}\phi_i(t) = \frac{1}{\mathbf{1}_N^T\mathbf{1}_N}\mathbf{1}_N^T\phi(t), \tag{1}$$

where $\phi(t) = [\phi_1(t), \dots, \phi_N(t)]^T$. Note that the dynamic average consensus will degenerate to a static average consensus problem once all reference signals remain constant instead of being time-varying.

C. Dynamic Weighted Average Consensus (DWAC)

The aforementioned average consensus is called the simple average consensus because the tracking consensus objective is an absolute average value of all the time-varying reference signals. However, the simple average might not be the required tracking objective in a practical scenario where each agent has its own weight on the reference signal. Therefore, we here mainly focus on a more general case that is also called the dynamic weighted average consensus, where each agent aims to track the following weighted average

$$\tilde{\phi}(t) = \frac{1}{\sum_{i=1}^{N} w_i}\sum_{i=1}^{N} w_i\phi_i(t) = \frac{1}{\mathbf{1}_N^T W\mathbf{1}_N}\mathbf{1}_N^T W\phi(t), \tag{2}$$

where $0 < w_i < 1, \forall i \in \{1, \dots, N\}$ is the normalized weight of agent i, $W = \mathrm{diag}(w_1, \dots, w_N)$ is a diagonal weight matrix. Note that the simple average $\bar{\phi}(t)$ is a special case of the weighted average $\tilde{\phi}(t)$. Specifically, the weighted average $\tilde{\phi}(t)$ is equivalent to the simple average $\bar{\phi}(t)$ when all agents in a network have the same weight (for example, $w_1 = \cdots = w_N = 1/N$).

Assumption 1: For each agent $i \in \{1, \dots, N\}$, $\phi_i(t)$ and $\dot{\phi}_i(t)$ are both bounded, i.e., there exist positive constants φ and ϱ such that

$$\begin{aligned} &\sup_{t\in[t_0,\infty)}||\phi(t)||_\infty \leq \varphi < \infty, \\ &\sup_{t\in[t_0,\infty)}||\dot{\phi}(t)||_\infty \leq \varrho < \infty, \end{aligned} \tag{3}$$

where $\dot{\phi}(t) = [\dot{\phi}_1(t), \dots, \dot{\phi}_N(t)]^T$ is the time derivative of the reference signal $\phi(t)$.

Remark 1: Note that the bounded time derivative of the time-varying reference signal implies that the input reference signal should not be changing too rapidly. Besides, we assume that the input reference signal of each agent is measurable and bounded.

III. Robust Dynamic Weighted Average Consensus Algorithm

To solve the dynamic weighted average consensus problem mentioned in the above section, we present a robust dynamic weighted average consensus (RDWAC) algorithm as follows

$$\dot{z}_i(t) = -\gamma z_i(t) + u_i(t), \tag{4a}$$

$$u_i(t) = -\alpha\sum_{j\in\mathcal{N}_i}\mathrm{sgn}\{x_i(t) - x_j(t)\}, \tag{4b}$$

$$x_i(t) = w_i^{-1} z_i(t) + \phi_i(t), \tag{4c}$$

where $z_i(t)$, $x_i(t)$ and $u_i(t)$ denote the internal state, the estimator state and the control input, respectively. The parameter $0 < w_i < 1, \forall i \in \{1, \dots, N\}$ is the normalized weight of agent i. The parameter α is the control gain and $\gamma > 0$ is a design parameter.

In a matrix form, the Algorithm (4) can be written as

$$\dot{z}(t) = -\gamma z(t) + u(t), \tag{5a}$$

$$u(t) = -\frac{\alpha}{2} B\text{sgn}\{B^T x(t)\}, \tag{5b}$$

$$x(t) = W^{-1} z(t) + \phi(t), \tag{5c}$$

where $x(t) = (x_1(t), \dots, x_N(t))^T, z(t) = (z_1(t), \dots, z_N(t))^T$, $u(t) = (u_1(t), \dots, u_N(t))^T, \phi(t) = (\phi_1(t), \dots, \phi_N(t))^T$, and B is the incidence matrix of the multi-agent network.

Note that the proposed Algorithm (4) can be seen as an extension of the works in [8], [18]. Compared to the recent work in [18], the differences of our proposed algorithm are at least two-fold. First, the control gain α is simple and fixed rather than complicated and time-varying. In contrast, the adaptive control gain in [18] needs more computation and memory resources because the update of the control gain is real-time and the amount of control gains of each agent might be multiple. Second, our proposed algorithm mainly focuses on the weighted average consensus problem which additionally introduces a weight w_i for each agent i and thus has a more general application potential.

Proposition 1: Given the robust dynamic weighted average consensus (RDWAC) Algorithm (4), if all agents' estimators $x_i(t), i \in \{1, \dots, N\}$ reach an agreement, then we have

$$\lim_{t\to\infty} x(t) = \tilde{\phi}(t)\mathbf{1}_N, \tag{6}$$

where $\tilde{\phi}(t) = \frac{1}{\mathbf{1}_N^T W \mathbf{1}_N}\mathbf{1}_N^T W\phi(t)$ is the weighted average of all the time-varying reference signals.

Proof: Let $p(t) = \mathbf{1}_N^T z(t)$. Taking the derivative of $p(t)$ with respect to time t, we have

$$\dot{p}(t) = -\gamma \mathbf{1}_N^T z(t) + \mathbf{1}_N^T u(t) = -\gamma p(t), \tag{7}$$

which implies that $p(t)$ exponentially decays to zero.

Since $\lim_{t\to\infty} p(t) = 0$, from Eq. (5c), we have $\lim_{t\to\infty} \mathbf{1}_N^T W x(t) = \mathbf{1}_N^T W\phi(t)$. Once all estimators $x_i(t)$ reach an agreement, then for $\forall i \in \{1, \dots, N\}$, we have $\mathbf{1}_N^T W \mathbf{1}_N \lim_{t\to\infty} x_i(t) = \mathbf{1}_N^T W\phi(t)$. Then we obtain

$$\lim_{t\to\infty} x_i(t) = \frac{1}{\mathbf{1}_N^T W \mathbf{1}_N}\mathbf{1}_N^T W\phi(t) = \tilde{\phi}(t), \tag{8}$$

which is equivalent to Eq. (6). The proof is completed. ■

Remark 2: Regarding the convergence of simple or weighted average consensus algorithms, the most important design is to ensure that the term $p(t) = \mathbf{1}_N^T z(t)$ always equals to 0 or tends to 0 as time goes on. In [10], the initialization condition makes $p(t)$ equal to 0 at all time, which plays a key role in achieving zero steady-state error. However, the specific initialization condition will fail when some agents join or leave the communication network, which implies that all agents in a network have to suffer from a cost of reinitialization after the network communication disruption in order to ensure zero steady-state error. In contrast, another viewpoint different from making $p(t)$ equal to 0 can be found in works [8], [18], where the leakage term $-\gamma z(t)$ makes all impacts of $z(t)$ to $x(t)$ decay exponentially to zero regardless of the initialization condition.

Theorem 1: For a connected undirected network, under Assumption 1, the robust dynamic weighted average consensus (RDWAC) Algorithm (4) guarantees that all estimates $x_i(t)$ exponentially converge to the weighted average of the time-varying reference signals $\phi(t)$ with any initial condition of $z(t)$ when the control gain α is chosen as

$$\alpha \geq \frac{2\sqrt{2N}\hat{w}(\varrho + \gamma\varphi) + \sqrt{\hat{w}}}{\sqrt{\lambda_2}}, \tag{9}$$

where $\hat{w} = \max\{w_1, \dots, w_N\}$. Furthermore, the accurate convergence with zero steady-state error is reached and remained for all $t \geq t^*$, where

$$t^* = \frac{1}{\gamma}\ln(\sqrt{2}\gamma||W^{\frac{1}{2}}e(t_0)||_2 + 1), \tag{10}$$

where $e(t_0) = x(t_0) - \tilde{\phi}(t_0)\mathbf{1}_N$ is the initial steady-state error.

Proof: Define the steady-state error $e(t) = x(t) - \tilde{\phi}(t)\mathbf{1}_N$. Due to the fact $B^T\mathbf{1}_N = 0$, we have $B^T e(t) = B^T(x(t) - \tilde{\phi}(t)\mathbf{1}_N) = B^T x(t)$. Combined with (5c), $e(t)$ can be written as

$$e(t) = W^{-1}z(t) + P\phi(t), \tag{11}$$

where $P = I_N - \frac{\mathbf{1}_N\mathbf{1}_N^T W}{\mathbf{1}_N^T W \mathbf{1}_N}$. Taking the derivative of $e(t)$ with respect to time t yields

$$\begin{aligned}\dot{e}(t) &= W^{-1}\dot{z}(t) + P\dot{\phi}(t)\\ &= -\gamma W^{-1}z(t) + W^{-1}u(t) + P\dot{\phi}(t).\end{aligned}$$

Adding and subtracting $\gamma P\phi(t)$, we further have

$$\dot{e}(t) = -\gamma e(t) + W^{-1}u(t) + P[\dot{\phi}(t) + \gamma\phi(t)]. \tag{12}$$

Define a quadratic Lyapunov function $V(t) = \frac{1}{2}e^T(t)We(t)$. The derivative of $V(t)$ is given by

$$\begin{aligned}\dot{V}(t) = &-\gamma e^T(t)We(t) - \frac{\alpha}{2}e^T(t)B\text{sgn}\{B^T x(t)\}\\ &+ e^T(t)WP\big[\dot{\phi}(t) + \gamma\phi(t)\big]. \end{aligned}\tag{13}$$

Note that

$$\begin{aligned}&-\frac{\alpha}{2}e^T(t)B\text{sgn}\{B^T x(t)\} = -\frac{\alpha}{2}||e^T(t)B||_1\\ &\leq -\frac{\alpha}{2}||e^T(t)B||_2 \leq -\frac{\alpha}{2}\sqrt{2\lambda_2}\sqrt{e^T(t)e(t)}\\ &\leq -\alpha\sqrt{\frac{\lambda_2}{\hat{w}}}\sqrt{V(t)},\\ &e^T(t)WP\big[\dot{\phi}(t) + \gamma\phi(t)\big] = e^T(t)W^{\frac{1}{2}}W^{\frac{1}{2}}P\big[\dot{\phi}(t) + \gamma\phi(t)\big]\\ &\leq \sqrt{N}||e^T(t)W^{\frac{1}{2}}||_2||W^{\frac{1}{2}}||_\infty||P||_\infty(||\dot{\phi}(t)||_\infty + \gamma||\phi(t)||_\infty)\\ &= \sqrt{2N}\sqrt{V(t)}||W^{\frac{1}{2}}||_\infty||P||_\infty(||\dot{\phi}(t)||_\infty + \gamma||\phi(t)||_\infty).\end{aligned}$$

Since $||W^{\frac{1}{2}}||_\infty = \sqrt{\hat{w}}$, $||P||_\infty \leq 2$, $||\phi(t)||_\infty \leq \varphi$, $||\dot{\phi}(t)||_\infty \leq \varrho$, we have

$$\begin{aligned}\dot{V}(t) &\leq -2\gamma V(t) - \alpha\sqrt{\frac{\lambda_2}{\hat{w}}}\sqrt{V(t)} + 2\sqrt{2}\sqrt{N\hat{w}}(\varrho + \gamma\varphi)\sqrt{V(t)}\\ &= -2\gamma V(t) + \left[2\sqrt{2}\sqrt{N\hat{w}}(\varrho + \gamma\varphi) - \alpha\sqrt{\frac{\lambda_2}{\hat{w}}}\right]\sqrt{V(t)}.\end{aligned}$$

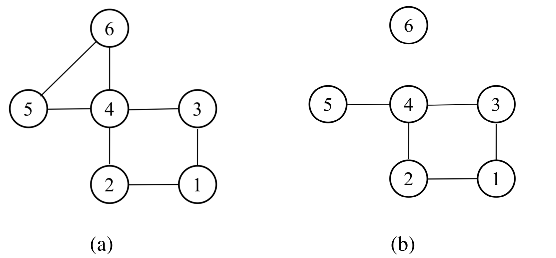

Fig. 1. Network topologies with 6 agents. (a) The initial network topology. (b) The agent 6 leaves the network.

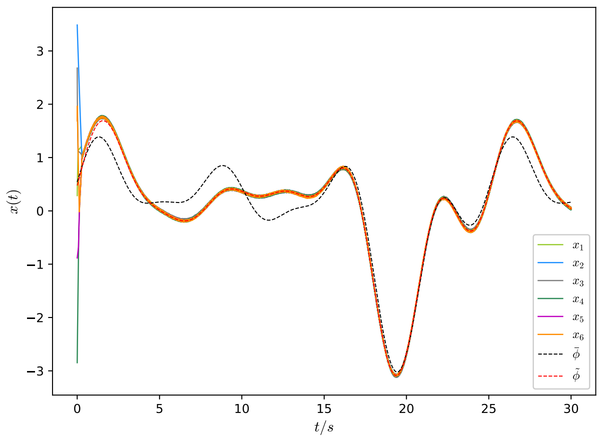

Fig. 2. The weighted average signal $\tilde{\phi}(t)$ is tracked using the proposed RDWAC algorithm.

Therefore, (9) yields

$$\dot{V}(t) \leq -2\gamma V(t) - \sqrt{V(t)}. \tag{14}$$

Solving the inequality (14) based on the comparison lemma ([20, Lemma 3.4]), we have

$$\sqrt{V(t)} \leq Ce^{-\gamma t} - \frac{1}{2\gamma}, \tag{15}$$

where $C = \frac{\sqrt{2}}{2}||W^{\frac{1}{2}}e(t_0)||_2 + \frac{1}{2\gamma}$. Since $V(t)$ is quadratic and $\dot{V}(t)$ is negative, we obtain that $e(t) = \mathbf{0}_N$ for all $t \geq t^*$, where

$$t^* = \frac{\ln(\sqrt{2}\gamma||W^{\frac{1}{2}}e(t_0)||_2 + 1)}{\gamma}. \tag{16}$$

The proof is completed. ■

Remark 3: Note that the above convergence analysis extends the results in [8], [18]. Specifically, a different Lyapunov function is constructed to solve the weighted dynamic average consensus instead of the simple dynamic average consensus. Furthermore, we here give a finite-time convergence analysis while recent works in [8], [18] only provide an asymptotic convergence analysis, which implies that the required convergence time is finite in this brief compared with the infinite convergence time in [8], [18]. Also, we give the lower bound of the required convergence time and establish the relationship between the lower bound and the initial steady-state error and control parameters explicitly. It can be seen that all estimators $x_i(t)$ converge to the weighted average consensus exponentially and the lower bound t^* can be adjusted by choosing a suitable design parameter γ.

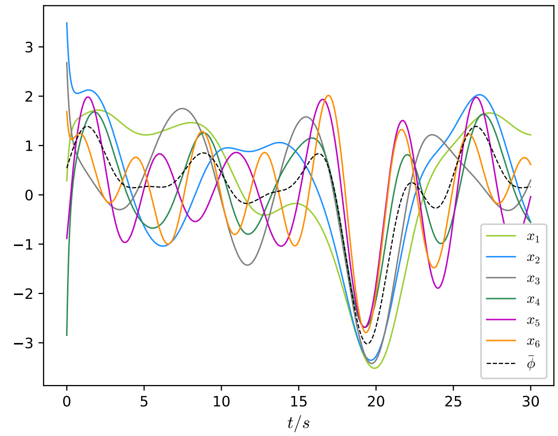

Fig. 3. The accurate signal tracking fails using the proportional DAC algorithm [8].

Remark 4: The lower bound in (9) depends on the global information of the reference signals and their time derivatives. It is worth noting that though the adaptive control gain $K(t)$ in [18] seems local and distributed, it still inevitably needs the information on the upper bounds of all reference signals and their time derivatives implicitly because a threshold value has to be computed to prevent the adaptive gain from drifting higher and higher. To the best of our knowledge, this issue is still open, which is worth further studying in the future work. On the other hand, the upper bounds of the global reference signals and their time derivatives can be obtained alternatively by implementing a maximum consensus algorithm in practice.

IV. SIMULATIONS

In this section, we provide some numerical examples to validate the RDWAC algorithm. First, we consider the scenario that the network topology remains unchanged which is given in Fig. 1(a). The reference signals $\phi_i(t)$ are selected as

$$\phi_i(t) = \begin{cases} a_i \sin(\omega_i t + \psi_i), & \forall i \in \{1, 2, 3\}, \\ a_i \cos(\omega_i t + \psi_i), & \forall i \in \{4, 5, 6\}, \end{cases} \tag{17}$$

where $a_i = \frac{i-1}{2} - 4$, $\omega_i = \frac{i+1}{4}$, and $\psi_i = \frac{2\pi i}{N} - \pi$. The initial values of the internal states $z_i(t)$ are generated in the range of $[0, 1]$ randomly. The normalized weight matrix W is chosen as $\text{diag}(0.16, 0.22, 0.10, 0.18, 0.20, 0.14)$. The control gain α and the design parameter γ in the RDWAC algorithm are set as 6.5 and 0.75 according to the lower bound in Theorem 1.

Some comparisons on the execution of our RDWAC algorithm and the algorithms in [8] and [18] are provided. Fig. 2 and Fig. 4 show that they can both asymptotically converge to the tracking signal as time goes on and the difference is that the estimators in Fig. 2 follow the weighted average signal $\tilde{\phi}(t)$ while those in Fig. 4 follow the absolute average signal $\bar{\phi}(t)$. Fig. 3 indicates that the proportional DAC algorithm cannot reach an agreement under the same parameter settings.

Furthermore, another scenario that some agents leave the interaction network midway and then join again is considered. Specifically, the initial network topology is given as Fig. 1(a); then at time $t = 5s$, the agent 6 leaves the interaction network, as shown in Fig. 1(b); finally at time $t = 11s$, the agent

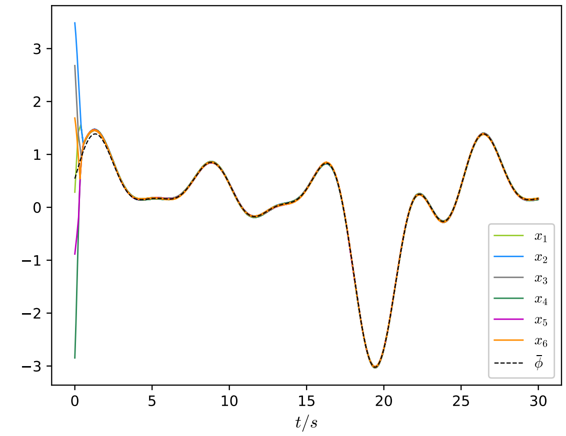

Fig. 4. The absolute average signal $\bar{\phi}(t)$ is tracked using the robust DAC algorithm [18].

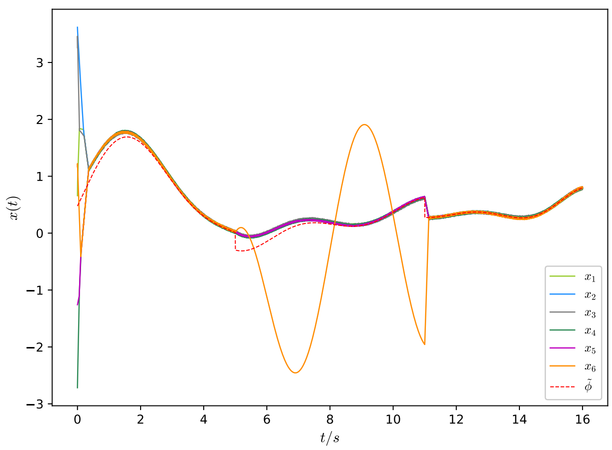

Fig. 5. The state evolution of estimators in the RDWAC algorithm with an agent leaving and joining the interaction network.

6 returns. In this case, the state evolution of the proposed RDWAC algorithm is shown in Fig. 5, from which we can see that the RDWAC algorithm is robust to the addition or reduction of cooperative agents.

V. Conclusion

In this brief, we proposed a robust dynamic weighted average consensus (RDWAC) algorithm that employs a simple fixed control gain and introduces a weight parameter for each agent such that the proposed algorithm can track a customized consensus objective instead of the simple average of the time-varying reference signals. Future works include the extension in the cases of general directed topologies and discrete-time communication.

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