Robust Distributed Average Tracking With Disturbance Observer Control

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***Abstract*— This paper is concerned with the study of robust distributed average tracking (DAT) algorithms for networked control systems in the presence of external disturbances or false data injection attacks (FDIAs). To eliminate the impacts of external disturbances and FDIAs, the technologies of disturbance-observer-based control (DOBC) and active disturbance rejection control (ADRC) are introduced into the context of DAT problems. First, for a class of external disturbances with known dynamics, we propose an anti-disturbance DAT (AD-DAT) algorithm, where a stand-alone disturbance observer based on the idea of DOBC is employed to estimate the disturbance and then to compensate it in the design of control inputs. The proposed AD-DAT algorithm can track the average of multiple time-varying reference signals with zero steady-state error and the accurate tracking is robust with respect to initialization constraints. Furthermore, for another class of FDIAs with unknown dynamics, we design an anti-attack DAT (AA-DAT) algorithm where the control input is based on the estimates of states instead of original states, and construct an extended state observer including a state observer and an FDIAs observer based on the idea of ADRC. The extended state observer plays a key role in estimating and eliminating the impact of FDIAs without compromising the accurate tracking performance. In addition, sufficient conditions are derived for the proposed two algorithms from a theoretical point of view to guarantee accurate average tracking. Finally, some numerical examples are given to illustrate the validity and effectiveness of the proposed algorithms.**

***Note to Practitioners*—This paper is motivated by the problem of robust distributed average tracking (DAT) for the time-varying centroid of the formation of a group of autonomous vehicles. The problem arises in the scenario where two groups of unmanned ground vehicles (UGVs) and unmanned aerial vehicles (UAVs) perform a combined surveillance-reconnaissance mission (where the UAVs aim to provide aerial coverage and early warning against threats for the UGVs, as shown in Fig. 1) in an uncertain environment where the external disturbances might exist or the FDIAs might be launched by adversaries. Obviously, the tracking accuracy of the target signal will be compromised in the presence of external disturbances or FDIAs. However, most existing works for disturbance rejection mainly focused on the static average consensus rather than the dynamic one even though a few works mentioned the DAT problem with only considering the case of external disturbances. Based on this, we propose an AD-DAT algorithm and an AA-DAT algorithm for the DAT problem based on the ideas of DOBC and ADRC, respectively. Numerical examples show that the proposed algorithms are able to estimate and eliminate the impacts of the external disturbances and the FDIAs, which implies that the algorithms can be implemented in practical scenarios. In future research, we will extend the results to more general scenarios in the presence of external disturbances and FDIAs.**

***Index Terms*— Distributed average tracking, dynamic average consensus, disturbance observers, false data injection, disturbance-observer-based control, active disturbance rejection control.**

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I. Introduction

RECENT years have witnessed great advances in sensing, computing and communication technologies, which motivates the study of how to control a team of autonomous robots to complete complex tasks cooperatively. In a multi-robot system, each robot usually needs to track or estimate the global performance simultaneously through local communication in order to adjust its own motion to improve the global control performance [1]. For example, consider a combined surveillance-reconnaissance mission which involves two groups of unmanned vehicles (ground and aerial), as shown in Fig. 1. In such a scenario, the two groups of unmanned ground vehicles (UGVs) and unmanned aerial vehicles (UAVs) are sent to establish a safe corridor cooperatively through hostile areas. The UGVs move in formation to clear a path through this area and the UAVs provide aerial coverage and early warning against threats for the UGVs. To achieve this goal,

Fig. 1. Two groups of unmanned ground vehicles (UGVs) and unmanned aerial vehicles (UAVs) perform a combined surveillance-reconnaissance mission, where the UAVs aim to provide aerial coverage and early warning against threats for the UGVs.

one key point is to guarantee that the UAVs are able to track the time-varying centroid of the UGV formation so that the UAVs can adjust their flight orbits to fully cover the moving UGVs.

The aforementioned problem, typically known as *distributed average tracking*[1] (DAT), aims to develop a control algorithm that enables each agent to track the average of multiple time-varying reference signals through local interaction among agents. The first work of DAT is given in [2], where a dynamic consensus algorithm is proposed to estimate the average of dynamic inputs with nonzero steady-state error under prescribed initialization constraints. To eliminate the initialization constraints, two algorithms named proportional (P) and proportional-integral (PI) are developed in [3]. The PI is further developed in [4], where the dynamic consensus can be achieved with zero steady-state error for some special classes of reference signals (e.g., polynomial signals with known orders and sinusoidal signals with known frequencies). For general time-varying input signals with bounded derivatives, a discontinuous sign function is introduced in [5] to achieve accurate tracking with zero steady-state error in finite time. In order to achieve zero steady-state error as well as initialization robustness, two robust DAT algorithms for both undirected and directed networks are developed in [6]. The dynamic weighted average consensus algorithms with finite-time convergence are studied in [7] and [8]. An algorithm for robust exact dynamic consensus of high order is also proposed in [9]. For the study of discrete-time algorithms, a first-order and a higher-order algorithm are proposed to achieve the DAT with a nonzero steady-state error under prescribed initial conditions in [10] and [11]. A multi-stage discrete time and randomized algorithm with small and bounded error is proposed in [12] and the accelerated version is proposed in [13]. To achieve initialization robustness, a robust discrete-time DAT algorithm with an arbitrarily small steady-state error is proposed in [14] by introducing a time-varying sequence of damping factors. A robust nonlinear estimator for DAT is also proposed in [15] to achieve small steady-state errors for slowly-varying inputs with an improved convergence rate. Also, the DAT problem has gained numerous extended studies on the Euler-Lagrange dynamics [16], double-integrator systems [17], [18], [19], formation control [20], [21], event-triggered control [22], [23], [24], uncertain directed topologies [25], distributed optimizaition [26], and privacy preservation [27], [28].

However, most of existing works about DAT problems do not consider the impact of external disturbances and false data injection attacks (FDIAs) even though the disturbances and malicious attacks widely exist in practical networked systems. Generally speaking, if external disturbances are injected into the control input of a control system, the final control performance will be compromised. To solve this limitation, an intuitive idea is to estimate the disturbance and then to compensate it in the design of control inputs. Based on this idea, a method named DOBC is proposed in [29] and [30]. For multi-agent systems, although there are numerous works [31], [32], [33], [34], [35], [36], [37], [38], [39] that focus on the study of disturbance rejection, they cannot be directly applied to the DAT problem, or have to face strong assumptions and low robustness. In particular, the works [31], [32], [33], [36], [37], [38], [39] are concerned with disturbance rejection in the context of static consensus and leader-follower consensus, respectively, rather than the DAT problem in our work. The work [34] proposes a disturbance rejection scheme for DAT problem, however, there exist several limitations: 1) A strong assumption about the time-varying reference signal is required, which plays a key role in achieving zero steady-state error. 2) The derivative of the reference signal needs to be known in designing the anti-disturbance DAT algorithm, which might be impossible in practical applications. 3) The proposed anti-disturbance DAT algorithm is non-robust since the initialization of the internal state needs to satisfy some prescribed conditions. In [35], a state feedback controller and an observer-based feedback controller are proposed to reject the external disturbance for DAT problem. However, it is worth emphasizing that the time-varying reference signal mentioned in [35] is a class of prescribed reference inputs with specific dynamics models. Besides, another additional limitation is that the dynamics of both the reference inputs and the external disturbances have the same linear structure, except for the parameters. Also, the anti-disturbance control and event-triggered control for delayed stochastic systems can be found in [40] and [41]. On the other hand, the FDIAs that can be seen as a kind of deception attacks, aim to corrupt the original signal received or be utilized by objective systems. To reduce the impact of FDIAs, some passive methods are developed by adjusting the control gain or frequency in [42] and [43] and the active attack compensation methods are introduced in [44] and [45] to achieve bounded consensus. Besides, the FDIAs can be modeled as unknown external disturbances, which are further studied in [46] and [47] by employing an extended state observer based on the idea of ADRC in [48] to estimate both the local state and the FDIAs. Also, the event-based control against deception attacks for nonlinear complex networks and cyber-physical systems is mentioned in [49] and [50].

[1]It can also be called *dynamic average consensus* (DAC) in other literature. Here we use the term *distributed average tracking* (DAT) to emphasize the tracking nature of the problem.

Motivated by the aforementioned works, this work aims to overcome the impacts of external disturbances and FDIAs in DAT by using the ideas of DOBC and ADRC, respectively. Compared to the traditional DOBC and ADRC in the context of static consensus or leader-follower consensus, the target signal to be tracked in DAT is based on multiple time-varying reference signals, rather than a group of static signals (in static consensus) or a single prescribed time-varying signal (in leader-follower consensus), which implies that the real-time and accuracy requirements in DAT are higher than in static consensus and leader-follower consensus. Based on this, a sign function with nonlinear and discontinuous characteristics is introduced in the design of control inputs compared to the linear and continuous dynamics models in static consensus and leader-follower consensus. To overcome the difficulties caused by the nonlinear and discontinuous dynamics models and the adverse impacts of external disturbances or attacks, it is necessary to design appropriate system control inputs and disturbance observers to estimate and compensate for the disturbances or attacks. Also, it is necessary to provide a new feasibility analysis for the proposed solution.

The main contributions are summarized as follows. First, for a class of external disturbances with known dynamics, we propose an AD-DAT algorithm, in which a disturbance compensation term is introduced to offset the external disturbance. To estimate the external disturbance, a stand-alone disturbance observer is proposed based on the idea of DOBC. The proposed AD-DAT algorithm guarantees that the average of multiple time-varying reference signals can be tracked with zero steady-state error even though there exist external disturbances in the execution of the algorithm. Furthermore, for FDIAs launched by adversaries, the FDIAs are modeled as unknown external disturbances and the AA-DAT algorithm is proposed by utilizing the estimates of states instead of original states in the design of control inputs. The proposed AA-DAT algorithm is equipped with an extended state observer which includes a state observer and an FDIA observer so that the impact of FDIAs can be eliminated by introducing compensation terms in the design of control inputs. In addition, for the proposed two algorithms, asymptotic convergence analyses are given to guarantee accurate average tracking from a theoretical point of view. Finally, some numerical examples are provided to illustrate the validity of the proposed algorithms.

The paper is organized as follows. Section II declares some preliminaries and the problem formulation. Section III provides the detailed algorithm design and convergence analysis for the robust distributed average tracking with external disturbances. For the case of false data injection attacks, the algorithm design and convergence analysis are given in Section IV. Some numerical examples are provided in Section V to illustrate the proposed algorithms. Finally, the paper is concluded in Section VI.

II. Preliminaries and Problem Formulation

A. Notation

The set of real numbers and positive real numbers are denoted by $\mathbb{R}$ and $\mathbb{R}^+$, respectively. The set of $n \times m$-dimensional real matrices is denoted by $\mathbb{R}^{n\times m}$ and the set of n-dimensional real column vectors is denoted by $\mathbb{R}^n$. The identity matrix and zero matrix of $n \times n$ dimensions are indicated by I_n and O_n, respectively. The 1 vector and 0 vector containing N entries are denoted by 1_N and 0_N, respectively. The transposes of a vector v and a matrix M are denoted by v^T and M^T, respectively. For $p \in [1, \infty]$, the p-norm of a vector x is denoted by $||x||_p$ and $||\cdot||_p$ is often written as $||\cdot||$ in the case of $p = 2$. The cardinality of a set $\mathcal{S}$ is indicated as $|\mathcal{S}|$. Let $\text{sgn}\{\cdot\}$ denote the sign function and $\text{sgn}\{x\} \triangleq [\text{sgn}\{x_1\}, \ldots, \text{sgn}\{x_n\}]^T$ for $\forall x \in \mathbb{R}^n$. The $\mathcal{L}_\infty$-norm of a piecewise continuous and bounded vector function $f(t)$ is defined as $||f(t)||_\infty = \sup_{t\geq 0} ||f(t)|| < \infty$ and the space is denoted by $\mathcal{L}_\infty$. The $\mathcal{L}_p$-norm of a piecewise continuous function $f(t)$ is defined as $||f(t)||_p = (\int_0^\infty ||f(t)||^p dt)^{1/p} < \infty$ and the space is denoted by $\mathcal{L}_p$, where $1 \leq p < \infty$.

B. Graph Theory

A connected undirected graph with N nodes is denoted by $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where the node set and the edge set are denoted by $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathcal{E} = \mathcal{V} \times \mathcal{V}$, respectively. The neighbor set of node i and its cardinality are represented by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$ and $N_i = |\mathcal{N}_i|$, respectively. The adjacency matrix of $\mathcal{G}$ is defined as $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ with entries of $a_{ij} = a_{ji} = 1$ if $(v_i, v_j) \in \mathcal{E}$, and $a_{ij} = a_{ji} = 0$ otherwise. The Laplacian matrix of graph $\mathcal{G}$ is defined as $L = (l_{ij}) \in \mathbb{R}^{N\times N}$ with entries $l_{ij} = -a_{ij}, i \neq j$ and $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$. Here L is a symmetric positive semidefinite matrix and its eigenvalues can be represented by $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$ in ascending order. The incidence matrix of graph $\mathcal{G}$ is defined as $B = (b_{ij}) \in \{-1, 0, 1\}^{N\times \ell}$ with entries $b_{ij} = -1$ if edge e_j leaves node v_i, $b_{ij} = 1$ if edge e_j enters node v_i, and $b_{ij} = 0$ otherwise.

C. Distributed Average Tracking (DAT)

Considering a networked multi-agent system composed of N agents, the linear dynamics of each agent i is described as follows

$$\dot{x}_i(t) = Ex_i(t) + Fu_i(t), \quad i = 1, \ldots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^p$ is the state, $u_i(t) \in \mathbb{R}^q$ is the control input to be designed, $E \in \mathbb{R}^{p\times p}$ is the state matrix, and $F \in \mathbb{R}^{p\times q}$ is the input matrix.

Suppose that each agent i has a time-varying reference signal $\phi_i(t) \in \mathbb{R}^p$ and can only communicate with its neighbors. The objective of DAT is to design a distributed control input $u_i(t)$ for each agent i such that all agents are able to track the average of multiple time-varying reference signals (it can be denoted by $\bar{\phi}(t) = \frac{1}{N}\sum_{i=1}^{N} \phi_i(t)$) through local interaction among agents. Take the UAVs' formation control in Fig. 1 for example, the control objective of DAT can be described as follows

$$||x_i(t) - \frac{1}{N}\sum_{i=1}^{N} \phi_i(t) - b_i|| \to 0, \quad t \to \infty, \tag{2}$$

where $x_i(t)$ is the UAV i's position and b_i is the desired distance from the target $\bar{\phi}(t)$. Without loss of generality, however, it is often assumed that $b_i = 0, \forall i \in \{1, \ldots, N\}$. Therefore, if all agents finally track the target signal $\bar{\phi}(t)$, i.e.,

$$||x_i(t) - \frac{1}{N}\sum_{i=1}^{N}\phi_i(t)|| \to 0, \quad t \to \infty, \tag{3}$$

we call that the DAT problem is solved.

The DAT has various applications including distributed formation control (in coverage, surveillance, and patrolling applications), distributed state estimation (in environmental monitoring, fire detection, object tracking), distributed resource allocation (in power grids, optimal routing, and economic systems), and distributed convex optimization (in large-scale machine learning and deep learning) [51]. Depending on the practical application scenarios, the interested physical variables (e.g., position, attitude, power consumption, etc.) can be modeled as the time-varying reference signal $\phi_i(t)$. For the sake of simplicity, we suppose that the time-varying reference signal of each agent is a scalar, that is, $\phi_i(t) \in \mathbb{R}$. Note that the scalar case (i.e., $\phi_i(t) \in \mathbb{R}$) can be extended to multi-dimensional cases through the operation of Kronecker product.

To solve the DAT problem (3), the control input in (1) can be designed as follows

$$u_i(t) = -\alpha \sum_{j\in\mathcal{N}_i} \operatorname{sgn}\{x_i(t) - x_j(t)\} + \gamma\phi_i(t) + \dot{\phi}_i(t), \tag{4}$$

with $E = -\gamma I_p$, $F = I_q$, and $p = q = 1$ in (1). In practice, however, the derivative $\dot{\phi}_i(t)$ in (4) might be unavailable for agent i. Thus, to avoid the direct use of $\dot{\phi}_i(t)$, an internal state is introduced and a robust DAT algorithm is proposed in recent works [6], [7] as follows

$$\dot{z}_i(t) = -\gamma z_i(t) + u_i(t), \tag{5a}$$
$$x_i(t) = z_i(t) + \phi_i(t), \tag{5b}$$
$$u_i(t) = -\alpha \sum_{j\in\mathcal{N}_i} \operatorname{sgn}\{x_i(t) - x_j(t)\}, \tag{5c}$$

where $z_i(t)$, $\phi_i(t)$ and $u_i(t)$ are the internal state, the time-varying reference signal and the control input of agent i, respectively. The parameters α and $\gamma > 0$ denote the control gain and the design parameter, respectively.

It is worth emphasizing that the DAT algorithm (5) is equivalent to the system (1) and (4). The state $x_i(t)$ denotes the time-varying estimate of the instantaneous average signal $\bar{\phi}(t)$ in agent i. The objective is to make all the states $x_i(t), i \in \{1, \ldots, N\}$ finally track the target signal $\bar{\phi}(t)$, which implies that not only all $x_i(t), i \in \{1, \ldots, N\}$ achieve an agreement but also the agreement value must be the target signal $\bar{\phi}(t)$. Consequently, each agent has to exchange its local estimate $x_i(t)$ instead of its reference signal $\phi_i(t)$ with its neighbors via local interaction. Also, the use of the discontinuous sign function can speed up the convergence process and even contribute to finite-time convergence.

III. Robust Distributed Average Tracking With External Disturbances

In practice, many disturbances such as environmental disturbances (geographical, meteorological, electromagnetic, etc.), aerodynamic disturbances, servo-drive disturbances, and unmodeled dynamics and parametric variations widely exist in networked control systems. Not surprisingly, these disturbances might bring adverse impacts on the performance and even the stability of control systems. However, most existing DAT works preset an ideal condition and do not take into account the presence of external disturbances, which results in that the existing DAT algorithm (5) cannot successfully track the target signal $\bar{\phi}(t)$ due to the adverse effects of external disturbances. Thus, the rejection of external disturbances for DAT is a key control objective in networked control systems.

A. Algorithm Design

Since the external disturbances cannot be measured directly or are too expensive to measure, an intuitive idea is to estimate the disturbances (or the influence of the disturbances) using measurable variables, and then compensate for the influence of the disturbances based on the disturbance estimates in the design of the control input. Based on this idea, we propose an anti-disturbance DAT (AD-DAT) algorithm as follows

$$\dot{z}_i(t) = -\gamma z_i(t) + \bar{u}_i(t), \tag{6a}$$
$$\bar{u}_i(t) = u_i(t) + d_i(t), \tag{6b}$$
$$x_i(t) = z_i(t) + \phi_i(t), \tag{6c}$$
$$u_i(t) = -\alpha \sum_{j\in\mathcal{N}_i} \operatorname{sgn}\{x_i(t) - x_j(t)\} - \hat{d}_i(t), \tag{6d}$$

where $d_i(t)$ is the external disturbance applied to the control input $u_i(t)$ and the term $\bar{u}_i(t)$ can be seen as the disturbed control input. To compensate the impact of the external disturbance $d_i(t)$, an estimate term $\hat{d}_i(t)$ is introduced in the design of the control input $u_i(t)$.

Assumption 1: Suppose that the external disturbance is harmonic and is generated by the following linear exogenous systems

$$\begin{aligned}\dot{\xi}_i(t) &= A_i\xi_i(t),\\ d_i(t) &= C_i\xi_i(t),\end{aligned} \tag{7}$$

where $\xi_i(t) \in \mathbb{R}^s$ is the internal state, and A_i and C_i are known constant matrices with appropriate dimensions and the pair (A_i, C_i) is observable.

Remark 1: Note that the harmonic disturbance is a class of disturbances that exist widely in engineering. For a specific environment, the parameter matrices A_i and C_i that are corresponding to frequency and observability features, can often be obtained by practical experience.

Next, the key point is to design an estimator for $\hat{d}_i(t)$ to estimate the disturbance $d_i(t)$. Based on the idea of DOBC in [30], we propose a stand-alone disturbance observer as follows

$$\begin{aligned}\dot{w}_i(t) = (A_i - K_iC_i)\,&[K_iz_i(t) + w_i(t)]\\ &- K_i\left[-\gamma z_i(t) + u_i(t)\right],\end{aligned} \tag{8a}$$
$$\hat{\xi}_i(t) = K_iz_i(t) + w_i(t), \tag{8b}$$
$$\hat{d}_i(t) = C_i\hat{\xi}_i(t), \tag{8c}$$

where $K_i \in \mathbb{R}^s$ is the gain matrix, $w_i(t) \in \mathbb{R}^s$ is the internal state of the observer, $\hat{\xi}_i(t)$ and $\hat{d}_i(t)$ are the estimates of $\xi_i(t)$ and $d_i(t)$, respectively.

Remark 2: For the class of harmonic disturbances with known dynamics (7), the DOBC method can estimate the disturbance directly by designing the disturbance observer (8), and then compensate for the disturbance in the design of the control input (6d). Besides, the DOBC method not only performs good estimation of the disturbance but also keeps the design of system models separate from that of disturbance observers, which can contribute to the implementation of algorithms in practice.

B. Main Result and Convergence Analysis

Before giving the main result in this section, some necessary assumptions and lemmas are provided.

Assumption 2: For any two neighboring agents, the local difference in signals $\phi_i(t)$ and their derivatives $\dot{\phi}_i(t)$ are both bounded, i.e., there exist positive constants φ and ϱ such that

$$\sup_{\substack{t\in[t_0,\infty) \\ \forall i,j:(v_i,v_j)\in\mathcal{E}}} ||\phi_i(t) - \phi_j(t)||_\infty \le \varphi < \infty,$$
$$\sup_{\substack{t\in[t_0,\infty) \\ \forall i,j:(v_i,v_j)\in\mathcal{E}}} ||\dot{\phi}_i(t) - \dot{\phi}_j(t)||_\infty \le \varrho < \infty. \tag{9}$$

Remark 3: Note that the constraints in (9) can be written as $\sup_{t\in[t_0,\infty)} ||B^T\phi(t)||_\infty \le \varphi$ and $\sup_{t\in[t_0,\infty)} ||B^T\dot{\phi}(t)||_\infty \le \varrho$ in a vector form, where $\phi(t) = [\phi_1(t), \ldots, \phi_N(t)]^T$, $\dot{\phi}(t) = [\dot{\phi}_1(t), \ldots, \dot{\phi}_N(t)]^T$, and B is the incidence matrix. In practical physical systems, due to mechanical and electrical limitations, the reference signal and its derivative cannot become infinitely large, which implies that they are always bounded. It is worth pointing out that the Assumption 2 is less strict than assuming that both $\phi_i(t)$ and $\dot{\phi}_i(t)$ should be continuous and bounded. Some similar assumptions are made in the literature [5], [6], and [20].

Lemma 1: For a connected undirected network with N nodes, the Laplacian matrix L and the incidence matrix B satisfy

$$M = LL^\dagger = BB^T(BB^T)^\dagger = B(B^TB)^\dagger B^T \tag{10}$$

where $M = I_N - \frac{1}{N}1_N 1_N^T$ and $(\cdot)^\dagger$ denotes the generalized inverse. Also, we can have that $L = BB^T = B^TB$ and $||(B^TB)^\dagger||_\infty \le ||L^\dagger||_2 \le 1/\lambda_2$.

Proof: See the Lemma 3 in [52]. ■

Remark 4: Note that we here consider the oriented incidence matrix for a given undirected graph. The oriented incidence matrix of an undirected graph is the incidence matrix, in the sense of the corresponding directed graph, where each edge is set as an any orientation compared with the undirected graph. That is, in the column of one edge of the incidence matrix, there is one 1 in the row corresponding to one vertex and one -1 in the row corresponding to the other vertex, and all other rows are 0. More details can be found in [53] and an example is provided in the section of Simulation.

Theorem 1: Consider a connected undirected network with external disturbance (7). The proposed AD-DAT algorithm (6) with disturbance observer (8) under Assumption 1-2 guarantees that the DAT problem (3) can be solved with zero steady-state error for given design parameter $\gamma > 0$ when the control gain α and a group of gain matrices $\{K_1, \ldots, K_N\}$ are chosen such that

$$\alpha \ge \frac{1}{\lambda_2}(\varrho + \gamma\varphi), \quad \Psi = \begin{bmatrix} \Lambda & \Theta_1 \\ \Theta_1^T & \Omega_1 \end{bmatrix} < 0, \tag{11}$$

where $\Lambda = -\gamma I_N$, $\Theta_1 = \frac{1}{2}\Xi_C$, $\Omega_1 = \frac{1}{2}\left(\Xi_{A-KC} + \Xi_{A-KC}^T\right)$ with $\Xi_C = \text{diag}(C_1, \ldots, C_N)$, $\Xi_{A-KC} = \text{diag}(A_1 - K_1C_1, \ldots, A_N - K_NC_N)$.

Proof: In a compact form, the AD-DAT algorithm (6) can be rewritten as

$$\dot{z}(t) = -\gamma z(t) + u(t) + d(t), \tag{12a}$$
$$x(t) = z(t) + \phi(t), \tag{12b}$$
$$u(t) = -\alpha B \cdot \text{sgn}\{B^T x(t)\} - \hat{d}(t), \tag{12c}$$

where $z(t) = (z_1(t), \ldots, z_N(t))^T$, $u(t) = (u_1(t), \ldots, u_N(t))^T$, $x(t) = (x_1(t), \ldots, x_N(t))^T$, $d(t) = (d_1(t), \ldots, d_N(t))^T$, $\hat{d}(t) = (\hat{d}_1(t), \ldots, \hat{d}_N(t))^T$.

Define estimation error $e_i(t) = \xi_i(t) - \hat{\xi}_i(t)$. It follows that

$$\begin{aligned}
\dot{e}_i(t) &= A_i\xi_i(t) - K_i\dot{z}_i(t) - \dot{w}_i(t) \\
&= A_i\xi_i(t) - K_i\left[-\gamma z_i(t) + u_i(t) + d_i(t)\right] \\
&\quad -\left[(A_i - K_iC_i)(K_iz_i(t) + w_i(t)) - K_i(-\gamma z_i(t) + u_i(t))\right] \\
&= (A_i - K_iC_i)(\xi_i(t) - \hat{\xi}_i(t)) \\
&= (A_i - K_iC_i)e_i(t),
\end{aligned}$$

which can be written in a compact form

$$\dot{e}(t) = \Xi_{A-KC}e(t), \tag{13}$$

where $e(t) = (e_1^T(t), \ldots, e_N^T(t))^T$, $\Xi_{A-KC} = \text{diag}(A_1 - K_1C_1, \ldots, A_N - K_NC_N)$.

Define steady-state error $\varepsilon_i(t) = x_i(t) - \bar{\phi}(t)$, and we have

$$\varepsilon(t) = x(t) - 1_N\bar{\phi}(t) = z(t) + M\phi(t), \tag{14}$$

where $\varepsilon(t) = (\varepsilon_1(t), \ldots, \varepsilon_N(t))^T$, $M = I_N - \frac{1}{N}1_N1_N^T$. Then it follows that

$$\begin{aligned}
\dot{\varepsilon}(t) &= -\gamma z(t) + u(t) + d(t) + M\dot{\phi}(t) \\
&= -\gamma\left[z(t) + M\phi(t)\right] - \alpha B \cdot \text{sgn}\{B^T\varepsilon(t)\} \\
&\quad + M\left[\dot{\phi}(t) + \gamma\phi(t)\right] - \hat{d}(t) + d(t) \\
&= -\gamma\varepsilon(t) - \alpha B \cdot \text{sgn}\{B^T\varepsilon(t)\} \\
&\quad + M\left[\dot{\phi}(t) + \gamma\phi(t)\right] + \Xi_C e(t),
\end{aligned} \tag{15}$$

where $\Xi_C = \text{diag}(C_1, \ldots, C_N)$.

Consider the following Lyapunov function candidate

$$\begin{aligned}
V(t) &= V_1(t) + V_2(t) \\
&\triangleq \frac{1}{2}\varepsilon^T(t)\varepsilon(t) + \frac{1}{2}e^T(t)e(t).
\end{aligned} \tag{16}$$

According to Lemma 1 and Assumption 2, we have

$$\begin{aligned}
&\varepsilon^T(t)M\left[\dot{\phi}(t) + \gamma\phi(t)\right] \\
&= \varepsilon^T(t)B(B^TB)^\dagger B^T\left[\dot{\phi}(t) + \gamma\phi(t)\right]
\end{aligned}$$

$$\leq ||\varepsilon^T(t)B||_1||(B^TB)^\dagger||_\infty\left[||B^T\dot{\phi}(t)||_\infty+\gamma||B^T\phi(t)||_\infty\right]$$
$$\leq \frac{1}{\lambda_2}(\varrho+\gamma\varphi)||\varepsilon^T(t)B||_1 \tag{17}$$

and

$$-\alpha\varepsilon^T(t)B\cdot\text{sgn}\{B^T\varepsilon(t)\}=-\alpha||\varepsilon^T(t)B||_1. \tag{18}$$

Then combining (15)-(18) yields

$$\begin{aligned}\dot{V}_1(t)&\leq-\gamma\varepsilon^T(t)\varepsilon(t)-\alpha||\varepsilon^T(t)B||_1\\&\quad+\frac{1}{\lambda_2}(\varrho+\gamma\varphi)||\varepsilon^T(t)B||_1+\varepsilon^T(t)\Xi_Ce(t)\\&=-\gamma\varepsilon^T(t)\varepsilon(t)+\left[\frac{1}{\lambda_2}(\varrho+\gamma\varphi)-\alpha\right]||\varepsilon^T(t)B||_1\\&\quad+\varepsilon^T(t)\Xi_Ce(t).\end{aligned} \tag{19}$$

If α is selected such that

$$\alpha\geq\frac{1}{\lambda_2}(\varrho+\gamma\varphi), \tag{20}$$

then we have

$$\dot{V}_1(t)\leq-\gamma\varepsilon^T(t)\varepsilon(t)+\varepsilon^T(t)\Xi_Ce(t). \tag{21}$$

Finally, it follows that

$$\begin{aligned}\dot{V}(t)&=\dot{V}_1(t)+\dot{V}_2(t)\\&\leq-\gamma\varepsilon^T(t)\varepsilon(t)+\varepsilon^T(t)\Xi_Ce(t)+e^T(t)\dot{e}(t)\\&\leq-\gamma\varepsilon^T(t)\varepsilon(t)+\varepsilon^T(t)\Xi_Ce(t)\\&\quad+e^T(t)\Xi_{A-KC}e(t)\\&=y^T(t)\Psi y(t)\leq-2\lambda_{\min}(-\Psi)V(t),\end{aligned} \tag{22}$$

where

$$y(t)=\begin{bmatrix}\varepsilon(t)\\e(t)\end{bmatrix},\Psi=\begin{bmatrix}\Lambda&\Theta_1\\\Theta_1^T&\Omega_1\end{bmatrix} \tag{23}$$

with $\Lambda=-\gamma I_N$, $\Theta_1=\frac{1}{2}\Xi_C$, $\Omega_1=\frac{1}{2}\left(\Xi_{A-KC}+\Xi_{A-KC}^T\right)$.

According to the condition (11), we have $\dot{V}(t)\leq-2\lambda_{\min}(-\Psi)V(t)<0$, which implies that the steady-state error is able to converge to zero asymptotically. The proof is completed. ■

Remark 5: For the choice of the group of gain matrices $\{K_1,\ldots,K_N\}$, we can let $K_i=K,\forall i\in\{1,\ldots,N\}$ and then use linear matrix inequality methods [54] to get a feasible solution of the gain matrix K. Alternatively, the linear matrix inequality (LMI) toolbox in MATLAB can be called directly to solve the linear matrix inequality in (11). Note that the parameter matrices A_i, C_i of all agents are required in the computation of the feasible solution. To collect the parameter matrices A_i, C_i of each agent $i, i\in\{1,\ldots,N\}$, some gossip algorithms can be executed during initialization. Since the collection of parameter matrices $A_i, C_i, i\in\{1,\ldots,N\}$ is only a one-time operation, the additional computation cost is trivial and negligible.

IV. Robust Distributed Average Tracking With False Data Injection Attacks

In addition to the disturbances and uncertainties caused by the natural environment and internal controls, in a hostile scenario, malicious adversaries might launch false data injection attacks (FDIAs) on the control system. Different from external disturbance and uncertainties, the FDIAs aim to inject some false signals to corrupt the original signals so that the actuator unit makes an incorrect response. Thus, the rejection of FDIAs launched by malicious adversaries for DAT is also a key control objective in networked control systems.

A. Algorithm Design

It is worth noting that, from a technical point of view, the FDIAs can be seen as external disturbances and what we need to do is to estimate the FDIAs (or the influence of the FDIAs) and then compensate for them in the design of the control inputs. Thus, we propose an anti-attack DAT (AA-DAT) algorithm as follows

$$\dot{z}_i(t)=-\gamma z_i(t)+\tilde{u}_i(t), \tag{24a}$$
$$\tilde{u}_i(t)=u_i(t)+\delta_i(t), \tag{24b}$$
$$x_i(t)=z_i(t)+\phi_i(t), \tag{24c}$$
$$u_i(t)=-\alpha\sum_{j\in\mathcal{N}_i}\text{sgn}\{x_i(t)-x_j(t)\}-\hat{\delta}_i(t), \tag{24d}$$

where $\delta_i(t)$ denotes the injected false data applied to the control input $u_i(t)$ and the term $\tilde{u}_i(t)$ can be seen as the disturbed control input. To compensate the impact of the injected false data $\delta_i(t)$, an estimate term $\hat{\delta}_i(t)$ is introduced in the design of the control input $u_i(t)$.

Assumption 3: The injected false data $\delta_i(t)$ and its time derivative $\dot{\delta}_i(t)=h_i(t)$ satisfy: 1) $\delta_i(t)$ and $h_i(t)$ are both bounded; 2) $h_i(t)\in\mathcal{L}_2$.

Remark 6: Note that some intrinsic features of the FDIAs need to be considered compared with traditional external disturbances. Specifically, many external disturbances can be modeled as known dynamics according to specific operation environments. In contrast, the FDIAs are usually launched by malicious adversaries, which implies that we have less knowledge about the injected attack signals. In other words, we cannot obtain the dynamics of the FDIAs, and we even do not know that the FDIAs are launched or not. Therefore, we here assume that the dynamic models of the FDIAs are unknown. Moreover, $h_i(t)\in\mathcal{L}_2$ implies that $||h_i(t)||_2=\left(\int_0^\infty h_i^2(\tau)d\tau\right)^{1/2}$ exists and is finite. In other words, we here consider a type of FDIAs where the injected attack signal tends to be a constant as time goes on or the FDIAs will be ended in finite time. Some similar assumptions are made in the literature [46], [47].

Now we are in a position to design the observers of $\hat{x}_i(t)$ and $\hat{\delta}_i(t)$ as follows

$$\dot{\hat{z}}_i(t)=-\gamma\hat{z}_i(t)+u_i(t)+\hat{\delta}_i(t)-\beta_1\left[\hat{x}_i(t)-x_i(t)\right], \tag{25a}$$
$$\hat{x}_i(t)=\hat{z}_i(t)+\phi_i(t), \tag{25b}$$
$$\dot{\hat{\delta}}_i(t)=-\beta_2\left[\hat{x}_i(t)-x_i(t)\right], \tag{25c}$$

where $\beta_1, \beta_2 > 0$ are positive scalar control gains, and $\hat{x}_i(t)$ and $\hat{z}_i(t)$ are the estimates of $x_i(t)$ and $z_i(t)$ in (24), respectively.

Remark 7: Compared to the harmonic disturbances with known dynamics in (7), for the class of FDIAs with unknown dynamics, it is difficult to design an observer to estimate the FDIAs directly, which implies that the DOBC method used in (8) cannot be adopted in the case of FDIAs directly. Since the FDIAs definitely influence the system output, the attack must be observable from the output. Based on this idea, a special observer named the extended state observer (25) is developed to estimate the attack via the impact of the attack on the system output indirectly. This idea is called active disturbance rejection control (ADRC), which can be found in [46], [47], and [48].

Remark 8: Actually, the proposed AA-DAT algorithm can be applied to deal with both unmodeled disturbances and attacks, as long as the disturbances or attacks tend to be constant values over time. For the deceptive attack launched by malicious attackers, there are two reasons why the attack signals are assumed to be vanishing towards constants over time. First, from the point of view of attackers, they always try to make the consensus point of the DAT deviate from the expected target point and stay undetected so that the control system makes wrong control decisions. For example, the air escort formation should be deployed with point "A" as the center. To invalidate the air escort, the attackers can mislead the formation to be centered on a wrong point "B" instead of the correct point "A" by launching a deceptive attack. Second, the injected attack signal has a great impact on the convergence of the whole system. Once the dynamics of the attack signal is too complex, the convergence of the DAT might be destroyed, which implies that the behavior of the deceptive attack might be detected and some anti-attack strategies will be implemented. Thus, the FDIAs focus more on stealthily changing decision results rather than disrupting the entire decision-making process. In conclusion, regardless of disturbances or attacks, as long as they tend to be constant values, the proposed AA-DAT algorithm is applicable.

B. Main Result and Convergence Analysis

Before giving the main result in this section, some necessary lemmas are provided.

Lemma 2: If $f(t), \dot{f}(t) \in \mathcal{L}_\infty$ and $f(t) \in \mathcal{L}_p$ with $1 \le p < \infty$, then $\lim_{t\to\infty} f(t) = 0$.

Proof: See the Lemma 3.2.5 in [55]. ■

Theorem 2: Consider a connected undirected network with FDIAs. The proposed AA-DAT algorithm (24) with state observer (25) under Assumption 2 and 3 guarantees that the DAT problem (3) can be solved with zero steady-state error for given design parameter $\gamma > 0$ if the control gains α, β_1, β_2, and a symmetric matrix $P > 0$ are chosen such that

$$\alpha \ge \frac{1}{\lambda_2}(\varrho + \gamma\varphi), \quad \Psi_1 = \begin{bmatrix} \Lambda & \Theta_2 & O_N \\ \Theta_2^T & \Omega_3 & \Gamma \\ O_N & \Gamma^T & -I_N \end{bmatrix} < 0 \tag{26}$$

with $\Lambda = -\gamma I_N$, $\Theta_2 = -\frac{1}{2}\bar{C}$, $\Omega_3 = \frac{1}{2}(P\bar{A} + \bar{A}^T P)$, $\Gamma = \frac{\sqrt{2}}{2} PQ$, where

$$\bar{A} = \begin{bmatrix} -(\gamma + \beta_1) I_N & I_N \\ -\beta_2 I_N & O_N \end{bmatrix},$$
$$\bar{C} = \begin{bmatrix} O_N & I_N \end{bmatrix}, \quad Q = \begin{bmatrix} O_N & -I_N \end{bmatrix}^T. \tag{27}$$

Proof: In a compact form, the proposed AA-DAT algorithm (24) can be rewritten as

$$\dot{z}(t) = -\gamma z(t) + u(t) + \delta(t), \tag{28a}$$
$$x(t) = z(t) + \phi(t), \tag{28b}$$
$$u(t) = -\alpha B \cdot \mathrm{sgn}\{B^T x(t)\} - \hat{\delta}(t), \tag{28c}$$

where $\delta(t) = (\delta_1(t), \ldots, \delta_N(t))^T$, $\hat{\delta}(t) = (\hat{\delta}_1(t), \ldots, \hat{\delta}_N(t))^T$.

Define estimation errors $e_{x_i}(t) = \hat{x}_i(t) - x_i(t)$ and $e_{\delta_i}(t) = \hat{\delta}_i(t) - \delta_i(t)$, then we have

$$e_x(t) = (e_{x_1}(t), \ldots, e_{x_N}(t))^T = \hat{x}(t) - x(t),$$
$$e_\delta(t) = (e_{\delta_1}(t), \ldots, e_{\delta_N}(t))^T = \hat{\delta}(t) - \delta(t),$$

where $\hat{x}(t) = (\hat{x}_1(t), \ldots, \hat{x}_N(t))^T$. Define steady-state error $\varepsilon_i(t) = x_i(t) - \bar{\phi}(t)$, and we have

$$\varepsilon(t) = x(t) - 1_N\bar{\phi}(t) = z(t) + M\phi(t). \tag{29}$$

Then it follows that

$$\begin{aligned}\dot{\varepsilon}(t) &= -\gamma z(t) + u(t) + \delta(t) + M\dot{\phi}(t)\\ &= -\gamma\left[z(t) + M\phi(t)\right] + u(t)\\ &\quad + M\left[\dot{\phi}(t) + \gamma\phi(t)\right] + \delta(t)\\ &= -\gamma\varepsilon(t) + u(t) + M\left[\dot{\phi}(t) + \gamma\phi(t)\right] + \delta(t).\end{aligned} \tag{30}$$

Consider the following Lyapunov function candidate

$$\begin{aligned}V(t) &= V_1(t) + V_2(t)\\ &\triangleq \frac{1}{2}\varepsilon^T(t)\varepsilon(t) + \frac{1}{2}e^T(t)Pe(t),\end{aligned} \tag{31}$$

where $e(t) = [e_x^T(t),\ e_\delta^T(t)]^T$, and $P > 0$ is a symmetric positive definite matrix. Using the similar derivation in (17), we have

$$\varepsilon^T(t)M\left[\dot{\phi}(t) + \gamma\phi(t)\right] \le \frac{1}{\lambda_2}(\varrho + \gamma\varphi)||\varepsilon^T(t)B||_1 \tag{32}$$

and

$$\begin{aligned}\varepsilon^T(t)u(t) &= -\alpha\varepsilon^T(t)B \cdot \mathrm{sgn}\{B^T x(t)\} - \varepsilon^T(t)\hat{\delta}(t)\\ &= -\alpha\varepsilon^T(t)B \cdot \mathrm{sgn}\{B^T[\varepsilon(t) + 1_N\bar{\phi}(t)]\} - \varepsilon^T(t)\hat{\delta}(t)\\ &= -\alpha||\varepsilon^T(t)B||_1 - \varepsilon^T(t)\hat{\delta}(t).\end{aligned} \tag{33}$$

Then combining (30)-(33) yields

$$\begin{aligned}\dot{V}_1(t) &= \varepsilon^T(t)\Big[-\gamma\varepsilon(t) - \alpha B \cdot \mathrm{sgn}\{B^T x(t)\}\\ &\qquad + M\left[\dot{\phi}(t) + \gamma\phi(t)\right] - e_\delta(t)\Big]\\ &\le -\gamma\varepsilon^T(t)\varepsilon(t) - \left[\alpha - \frac{1}{\lambda_2}(\varrho + \gamma\varphi)\right]||\varepsilon^T(t)B||_1\\ &\quad - \varepsilon^T(t)\bar{C}e(t)\\ &\le -\gamma\varepsilon^T(t)\varepsilon(t) - \varepsilon^T(t)\bar{C}e(t),\end{aligned} \tag{34}$$

where $\bar{C}$ is as in (27), if the control gain α is selected such that

$$\alpha \geq \frac{1}{\lambda_2}(\varrho + \gamma\varphi). \tag{35}$$

Since

$$\dot{e}_x(t) = \dot{\hat{x}}(t) - \dot{x}(t) = -(\gamma + \beta_1)e_x(t) + e_\delta(t),$$
$$\dot{e}_\delta(t) = \dot{\hat{\delta}}(t) - \dot{\delta}(t) = -\beta_2 e_x(t) - h(t),$$

where $h(t) = (h_1(t), \ldots, h_N(t))^T$, we have

$$\dot{e}(t) = \bar{A}e(t) + Qh(t), \tag{36}$$

where

$$\bar{A} = \begin{bmatrix} -(\gamma + \beta_1)I_N & I_N \\ -\beta_2 I_N & O_N \end{bmatrix}, Q = \begin{bmatrix} O_N \\ -I_N \end{bmatrix}. \tag{37}$$

It follows that

$$\begin{aligned}\dot{V}_2(t) &= e^T(t)P\dot{e}(t) \\ &= e^T(t)P\left[\bar{A}e(t) + Qh(t)\right] \\ &\leq e^T(t)P\bar{A}e(t) + \frac{1}{2}e^T(t)PQQ^TPe(t) + \frac{1}{2}h^T(t)h(t),\end{aligned}$$

where the basic inequality $a^Tb \leq (a^Ta + b^Tb)/2$ is used. Thus, we have

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) \\ &\leq -\gamma\varepsilon^T(t)\varepsilon(t) - \varepsilon^T(t)\bar{C}e(t) \\ &\quad + e^T(t)P\bar{A}e(t) + \frac{1}{2}e^T(t)PQQ^TPe(t) + \frac{1}{2}h^T(t)h(t) \\ &= y^T(t)\Psi y(t) + \frac{1}{2}h^T(t)h(t),\end{aligned} \tag{38}$$

where

$$y(t) = \begin{bmatrix} \varepsilon(t) \\ e(t) \end{bmatrix}, \Psi = \begin{bmatrix} \Lambda & \Theta_2 \\ \Theta_2^T & \Omega_2 \end{bmatrix} \tag{39}$$

with $\Lambda = -\gamma I_N$, $\Theta_2 = -\frac{1}{2}\bar{C}$, $\Omega_2 = \frac{1}{2}(P\bar{A} + \bar{A}^TP + PQQ^TP)$.

From (31) and (39), we have

$$\frac{\check{\sigma}}{2}y^T(t)y(t) \leq V(t) \leq \frac{\hat{\sigma}}{2}y^T(t)y(t), \tag{40}$$

where $\check{\sigma} = \min\{1, \lambda_{\min}(P)\}$, $\hat{\sigma} = \max\{1, \lambda_{\max}(P)\}$. Thus, from (38), we have

$$\begin{aligned}\dot{V}(t) &\leq -\lambda_{\min}(-\Psi)y^T(t)y(t) + \frac{1}{2}h^T(t)h(t) \\ &\leq \frac{-2\lambda_{\min}(-\Psi)}{\check{\sigma}}V(t) + \frac{1}{2}h^T(t)h(t).\end{aligned} \tag{41}$$

Let $\Psi < 0$, which is equivalent to

$$\Psi_1 = \begin{bmatrix} \Lambda & \Theta_2 & O_N \\ \Theta_2^T & \Omega_3 & \Gamma \\ O_N & \Gamma^T & -I_N \end{bmatrix} < 0 \tag{42}$$

with $\Omega_3 = \frac{1}{2}(P\bar{A} + \bar{A}^TP)$, $\Gamma = \frac{\sqrt{2}}{2}PQ$ according to Schur complement lemma [56].

Integrating both sides of (41) yields

$$V(t) - V(0) \leq \frac{-2\lambda_{\min}(-\Psi)}{\check{\sigma}}\int_0^t V(\tau)d\tau + \frac{1}{2}\int_0^t h^T(\tau)h(\tau)d\tau. \tag{43}$$

Combining (40) and (43) yields

$$\lambda_{\min}(-\Psi)\int_0^t y^T(\tau)y(\tau)d\tau \leq V(0) + \frac{1}{2}\int_0^t h^T(\tau)h(\tau)d\tau. \tag{44}$$

Letting $t \to \infty$, we have

$$\lambda_{\min}(-\Psi)\int_0^\infty y^T(\tau)y(\tau)d\tau \leq V(0) + \frac{1}{2}\int_0^\infty h^T(\tau)h(\tau)d\tau. \tag{45}$$

Since Assumption 3 holds, it follows that $\int_0^\infty h^T(\tau)h(\tau)d\tau < \infty$ exists and is finite. Due to the fact that $V(0)$ is bounded under Assumptions 2 and 3, we have that $\int_0^\infty y^T(\tau)y(\tau)d\tau < \infty$ exists and is finite, which implies $y(t) \in \mathcal{L}_2$.

From (43), we can have

$$V(t) \leq V(0) + \frac{1}{2}\int_0^\infty h^T(\tau)h(\tau)d\tau, \tag{46}$$

which implies $V(t), \dot{V}(t) \in \mathcal{L}_\infty$ under Assumption 3. Then it follows that $y(t), \dot{y}(t) \in \mathcal{L}_\infty$ based on (31) and (39). Thus, according to Lemma 2, we have $\lim_{t\to\infty} y(t) = 0$. Then it follows that the error systems $\varepsilon(t)$ and $e(t)$ are asymptotically stable, i.e.,

$$\lim_{t\to\infty}\varepsilon(t) = 0, \ \lim_{t\to\infty}e_x(t) = 0, \ \lim_{t\to\infty}e_\delta(t) = 0,$$

which implies that

$$\lim_{t\to\infty}x(t) = 1_N\bar{\phi}(t). \tag{47}$$

The proof is completed. ■

Remark 9: Note that the classical solution exists even though a discontinuous sign function is employed in (6) and (24) (see Example 6 in [57]). Furthermore, the considered Lyapunov functions (16) and (31) are both continuous and differentiable. Thus, the nonsmooth analysis methods for stability analysis are not required in this paper.

Remark 10: In control system design, a large proportion of control algorithms are designed in continuous time. In practice, to implement the continuous-time control algorithm, a discrete-time algorithm can be easily obtained by Euler discretization with a small step size h. When $h \to 0$, the stability and convergence properties of the discrete-time algorithm are expected to be similar to those of the continuous-time algorithm. For the analysis of the upper bound of the step size h, it is worth the effort to establish new theoretical results, which is beyond the scope of our current work.

Remark 11: Note that the discontinuous sign function in (6) and (24) might lead to chattering effect in practical implementation. To solve this issue, a continuous sigmoid function can be employed to approximate the discontinuous sign function. Furthermore, the control gain α in Theorem 1 and 2 needs to depend on parameters ϱ, φ and λ_2. For ϱ and φ, a maximum consensus algorithm can be employed to achieve the maximum operation. For λ_2, a conservative lower bound $\lambda_2 \geq 4/(N(N-1))$ (see Theorem 4.2 in [58]) can be used and other tighter bounds or distributed estimation methods for λ_2 can also be found in literature. In addition, the convergence rate is determined by the design parameter γ according to (22)-(23) and (41)-(42). Actually, a bigger γ tends to produce a bigger convergence rate, which is associated with a larger control gain α according to the parameter constraints in Theorem 1 and 2.

Remark 12: As we can see from (6)-(8) and (24)-(25), to achieve the accurate target tracking in the presence of external disturbances or attacks, both the control input and the disturbance observer are proposed for the AD-DAT and AA-DAT algorithms, respectively. In general, there might exist various disturbances, attacks and uncertainties in practical systems and the system dynamics might also be of high-order or in a general linear or nonlinear form. However, how to design appropriate control inputs and disturbance observers under such scenarios is still open and is worthy of further study.

V. Simulation

A. Robust Distributed Average Tracking With External Disturbances

In this subsection, we first consider a scenario where there exist external disturbances in the execution of DAT algorithms. Suppose that the network topology is composed of 6 agents and is shown as in Fig. 2. For the initial topology in Fig. 2(a), the Laplacian matrix and the oriented incidence matrix are given as follows

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 \\ 0 & -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix},$$

from which we can see that $L = BB^T$ in Lemma 1 holds.

For agent i, the time-varying reference signal $\phi_i(t)$ is set as

$$\phi_i(t) = \begin{cases} a_i \sin(\omega_i t + \psi_i), & \forall i \in \{1, 2, 3\}, \\ a_i \cos(\omega_i t + \psi_i), & \forall i \in \{4, 5, 6\}, \end{cases} \tag{48}$$

where $a_i = \frac{i-1}{2} - 4$, $\omega_i = \frac{i+1}{4}$, and $\psi_i = \frac{\pi i}{3} - \pi$. For the initial values of internal states $z_i(t), i \in \{1, \ldots, 6\}$, they are generated in the range of $[0, 1]$ randomly. The design parameter γ and control gain α are chosen as 0.75 and 8.5, respectively, according to Theorem 1.

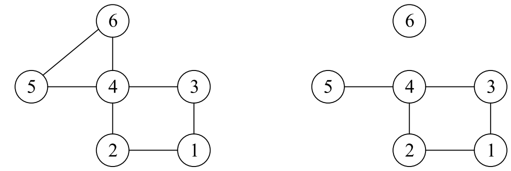


Fig. 2. The network topology composed of 6 agents. (a) The initial topology. (b) A different topology for the same problem where agent 6 has become disconnected.

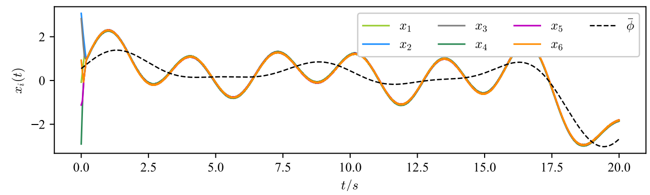


Fig. 3. The target signal $\bar{\phi}(t)$ fails to be tracked by executing the traditional DAT algorithm in [6] and [7] with external disturbances.

For the external disturbance in (7), we assume that the internal state $\xi_i(t)$ is two-dimensional and the parameters A_i, C_i are set as $A_i = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ and $C_i = [5, 0]$ for all agents. Then we solve the ordinary differential equation (7) by using MATLAB's library functions to obtain

$$d_i(t) = 2.5\cos(1)\sin(2t) + 2.5\sin(1)\cos(2t) \tag{49}$$

for given initial internal state $\xi_i(0) = [0.5\sin(1), 0.5\cos(1)]^T$. The gain matrix of the disturbance observer is set as $K_i = [18.5, 18.5]^T$ for $i \in \{1, \ldots, 6\}$ by applying LMI Solvers of MATLAB according to the parameter constraint (11) in Theorem 1.

For the first scenario, we consider that the network topology remains unchanged and the topology is shown in Fig. 2(a). When a traditional DAT algorithm is executed with external disturbances, the state evolution in Fig. 3 shows that the target signal $\bar{\phi}(t)$ denoted by a dashed black line fails to be tracked. In contrast, when the proposed AD-DAT algorithm (6) with disturbance observer (8) is executed, the state evolution in Fig. 4 shows that the target signal $\bar{\phi}(t)$ can be tracked successfully while all state estimates achieve an agreement asymptotically. The state evolution of the disturbance observer and the external disturbance are shown in Fig. 5, which implies that the external disturbance (7) can be tracked by using the proposed disturbance observer (8).

For another scenario, we consider that some agents leave the network and then rejoin. For example, let the agent 6 leave the network at time $t = 6s$ and then return at time $t = 15s$. In this case, the state evolution of all agents are shown in Fig. 6, which shows that the proposed AD-DAT algorithm is robust to the change of network topologies. In other words, all agents in a network are able to converge to the target signal $\bar{\phi}(t)$ adaptively without any reinitialization despite that some agents leave or join the network.

B. Robust Distributed Average Tracking With FDIAs

In this subsection, we consider another scenario where there exist FDIAs in the execution of DAT algorithms. According

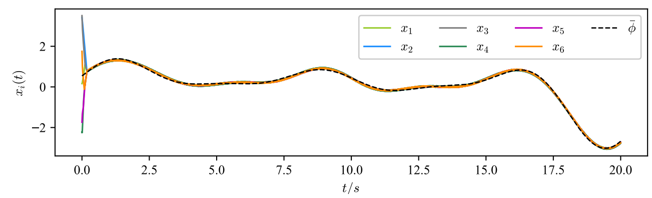


Fig. 4. The target signal $\bar{\phi}(t)$ can be tracked by executing the proposed AD-DAT algorithm.

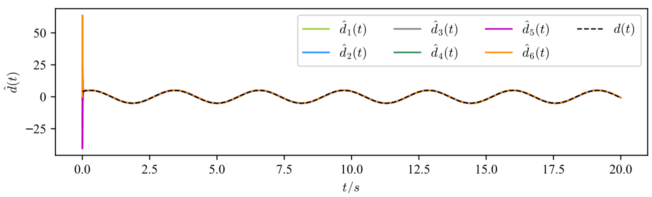


Fig. 5. The states of the external disturbance and the disturbance observer.

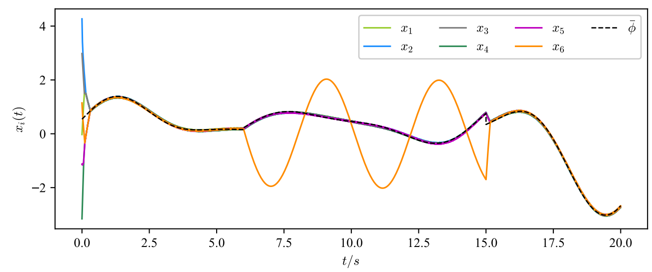


Fig. 6. The proposed AD-DAT algorithm is robust to the change of network topologies.

to Assumption 3, the unknown FDIAs can be set as

$$\delta_i(t) = a_i(1 - \cos(b_i t)\mathrm{e}^{-c_i t}), i \in \{1, \ldots, 6\}, \tag{50}$$

where $a_i = i - 1.5, b_i = (i + 2)/2, c_i = 0.5(i + 1)$ for agent i. The parameters α, γ remain the same as before. The control gains of the extended state observer can be set as $\beta_1 = 1.5, \beta_2 = 2.5$ by solving the linear matrix inequality (26) in Theorem 2 with LMI Solvers of MATLAB.

When a traditional DAT algorithm is executed with FDIAs, the state evolution in Fig. 7 shows that the target signal $\bar{\phi}(t)$ fails to be tracked due to the existence of FDIAs. In contrast, when the proposed AA-DAT algorithm (24) with the extended state observer (25) is executed, the state evolution in Fig. 8 shows that the target signal $\bar{\phi}(t)$ can be tracked while all state estimates achieve an agreement asymptotically. Furthermore, the estimation errors $e_{x_i}(t) = \hat{x}_i(t) - x_i(t)$ and $e_{\delta_i}(t) = \hat{\delta}_i(t) - \delta_i(t)$ are shown in Fig. 9, from which we can see that the estimation errors asymptotically converge to zero.

Note that the stability and convergence properties of the DAT module under external disturbances and attacks might be compromised because stability and zero error steady-state convergence of the DAT depends on the asymptotic compensation of the disturbances or attacks. As we can see, the convergence speed in Fig. 4 is much faster than that in Fig. 8. The main reason is that the convergence speeds of the two observers are different. In simulation part A, since the dynamics of the considered disturbance is known, the proposed observer (8) is able to estimate the disturbance directly, which implies that the convergence speed is faster. In simulation part B, since the dynamics of the considered attack is unknown, it is difficult to design an observer to estimate the attack itself directly. Instead, we have to propose an extended state observer (25) to estimate the impact of the attack on the system output indirectly. As a result, the convergence speed of the indirect estimation method is much slower than that of the direct estimation method used in the observer (8). In conclusion, the practical convergence properties of the DAT module under external disturbances and attacks are closely related to the estimation and compensation schemes of the disturbances or attacks.

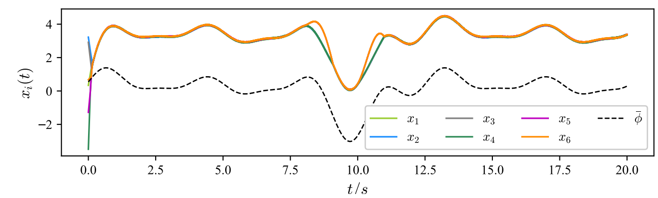


Fig. 7. The target signal $\bar{\phi}(t)$ fails to be tracked by executing the traditional DAT algorithm in [6] and [7] with FDIAs.

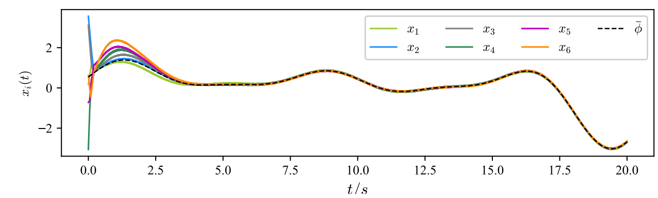


Fig. 8. The target signal $\bar{\phi}(t)$ can be tracked by executing the proposed AA-DAT algorithm.

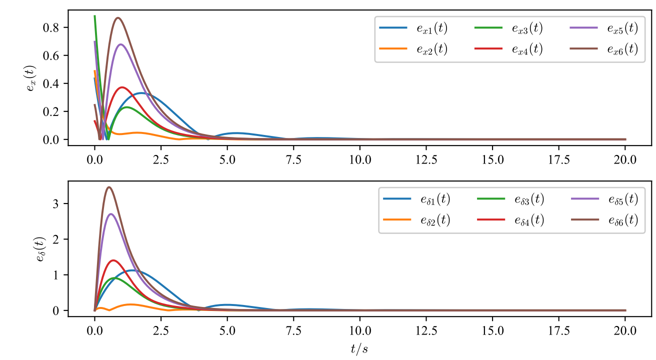


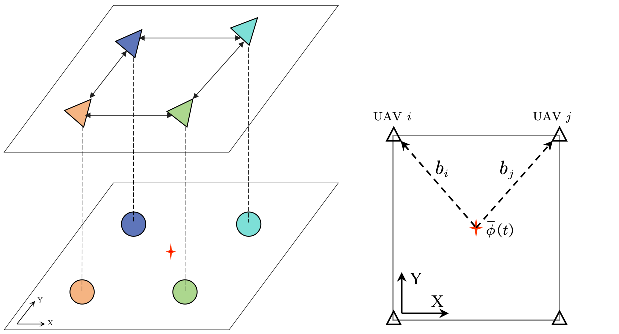
Fig. 9. The estimation errors of the proposed extended state observer (25).

C. Application to Distributed Formation Control

In this subsection, we will show how the DAT is applied to the formation control in a combined surveillance-reconnaissance mission shown in Fig. 1 in Section I.

Consider that there are four UGVs moving in formation and four UAVs flying above to provide aerial coverage and early warning. The objective of the UAVs is to track the time-varying geometric center of the UGVs while spreading out in a pre-specified formation. In this scenario, each UAV can monitor the position of a UGV and share relevant information with its neighbors via wireless communication, as shown in Fig. 10 (a). Since each UAV can only monitor one UGV, it needs to cooperate with its neighbors to compute the geometric center of the group of UGVs, which implies that a distributed control algorithm is required instead of a centralized one in the above scenario.

Each UAV i can access to its own position $p_i(t) \triangleq [p_{xi}(t),\ p_{yi}(t)]^T \in \mathbb{R}^2$ and monitor the UGV i's position $\phi_i(t) \triangleq [\phi_{xi}(t),\ \phi_{yi}(t)]^T \in \mathbb{R}^2$. The positions $p_i(t)$ and $\phi_i(t)$



(a) (b)

Fig. 10. (a) DAT based formation control. The triangles are the UAVs, the circles are the UGVs, and the cross is the center of the UGVs. (b) The relative positions between the UAVs and the time-varying geometric center of the UGVs.

are both expressed in the inertial coordinate frame. Note that we here only focus on the planar coordinates of the UAVs since the heights of all the UAVs are assumed to be the same. To spread out and maintain a pre-specified formation (assuming it is invariant), each UAV i must be driven to a relative position vector $b_i \triangleq [b_{xi},\ b_{yi}]^T \in \mathbb{R}^2$ (as shown in Fig. 10 (b)) with respect to the time-varying geometric center of the UGVs, i.e., $p_i(t) \to \frac{1}{N}\sum_{i=1}^{N}\phi_i(t) + b_i$ as $t \to \infty$. In this scenario, we assume that the time-varying position trajectories of the UGVs are given by

$$\phi_1(t) = \begin{bmatrix} \sin(0.2t) + 0.75t + 2.0 \\ -\sin(0.1t) + 0.4t + 1.0 \end{bmatrix},$$
$$\phi_2(t) = \begin{bmatrix} -\sin(0.3t) + 0.75t - 1.0 \\ \sin(0.2t) + 0.4t + 2.0 \end{bmatrix},$$
$$\phi_3(t) = \begin{bmatrix} -\sin(0.1t) + 0.75t - 2.0 \\ \sin(0.2t) + 0.4t - 1.0 \end{bmatrix},$$
$$\phi_4(t) = \begin{bmatrix} \sin(0.2t) + 0.75t + 1.0 \\ -\sin(0.3t) + 0.4t - 2.0 \end{bmatrix}.$$

The initial positions of the UAVs are set as $p_1(0) = [4.2,\ 3.8]^T$, $p_2(0) = [-2.0,\ 1.8]^T$, $p_3(0) = [-2.0,\ -1.0]^T$, $p_4(0) = [4.2,\ -1.5]^T$. Also, we assume that the pre-specified formation is a rectangle with the relative position vectors $b_1 = [4.0,\ 4.0]^T$, $b_2 = [-4.0,\ 4.0]^T$, $b_3 = [-4.0,\ -4.0]^T$, $b_4 = [4.0,\ -4.0]^T$.

We first consider the case of external disturbances. To achieve the formation control of the UAVs, we propose the following anti-disturbance DAT (AD-DAT) algorithm

$$\dot{z}_i(t) = -\gamma z_i(t) + \bar{u}_i(t), \tag{51a}$$
$$\bar{u}_i(t) = u_i(t) + d_i(t), \tag{51b}$$
$$p_i(t) = z_i(t) + \phi_i(t) + b_i, \tag{51c}$$
$$u_i(t) = -\alpha \sum_{j\in\mathcal{N}_i} \mathrm{sgn}\{p_i(t) - b_i - (p_j(t) - b_j)\} - \hat{d}_i(t), \tag{51d}$$

where $p_i(t), z_i(t), \phi_i(t)$ and b_i are all two-dimensional vectors now, and are respectively the position state of UAV i, the internal state of UAV i, the position state of UGV i and the relative position vector of UAV i. $d_i(t) \in \mathbb{R}^2$ is the external disturbance applied to the control input $u_i(t)$ and $\hat{d}_i(t)$ is the

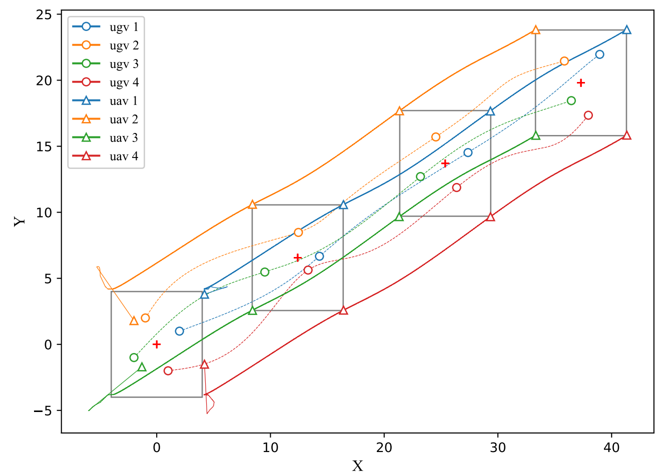


Fig. 11. The motion trajectories of the UGVs and the UAVs in the presence of external disturbances. The circles are the UGVs, the triangles are the UAVs, the crosses are the centers, and the rectangles are the pre-specified formation shapes of the UAVs.

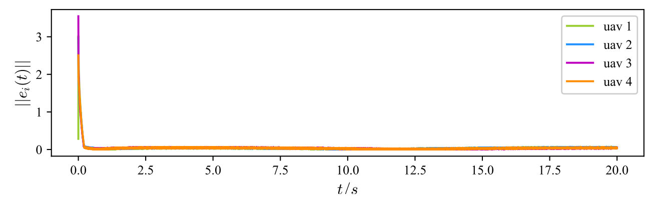


Fig. 12. The tracking errors of the UAVs in the presence of external disturbances.

estimate of $d_i(t)$. Note that the algorithm (51) is equivalent to the AD-DAT algorithm (6) in Section III when we denote $x_i(t) = p_i(t) - b_i$.

Assume that the external disturbance $d_i(t)$ is set as in (49) and is extended to a two-dimensional vector. For the initial values of internal states $z_i(t), i \in \{1, \ldots, 4\}$, they are generated in the range of $[0, 1]$ randomly. The design parameter γ, control gain α and the gain matrix K_i are the same as in Section V-A. By implementing the proposed AD-DAT algorithm (51) and the disturbance observer (8), the motion trajectories of the UGVs and the UAVs are shown in Fig. 11. We can see that the UAVs are able to track the time-varying geometric center of the UGVs and adjust their own positions to spread out and maintain a pre-specified formation shape. The tracking error $e_i(t) = p_i(t) - b_i - \frac{1}{N}\sum_{i=1}^{N}\phi_i(t)$ is shown in Fig. 12, from which we can see that the errors $e_i(t), \forall i \in \{1, \ldots, 4\}$ asymptotically converge to zero with small perturbations, which are due to the chatting effect of the discontinuous sign function.

Next, we consider the case of false data injection attacks (FDIAs). To achieve the formation control of the UAVs, we propose the following anti-attack DAT (AA-DAT) algorithm

$$\dot{z}_i(t) = -\gamma z_i(t) + \tilde{u}_i(t), \tag{52a}$$
$$\tilde{u}_i(t) = u_i(t) + \delta_i(t), \tag{52b}$$
$$p_i(t) = z_i(t) + \phi_i(t) + b_i, \tag{52c}$$
$$u_i(t) = -\alpha \sum_{j\in\mathcal{N}_i} \mathrm{sgn}\{p_i(t) - b_i - (p_j(t) - b_j)\} - \hat{\delta}_i(t), \tag{52d}$$

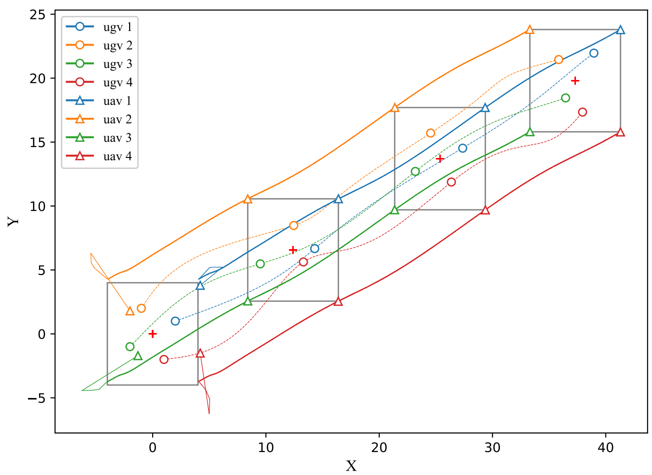


Fig. 13. The motion trajectories of the UGVs and the UAVs in the presence of FDIAs. The circles are the UGVs, the triangles are the UAVs, the crosses are the centers, and the rectangles are the pre-specified formation shapes of the UAVs.

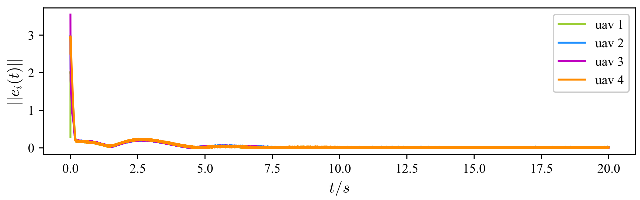


Fig. 14. The tracking errors of the UAVs in the presence of FDIAs.

where $\delta_i(t) \in \mathbb{R}^2$ is the injected false data applied to the control input $u_i(t)$ and $\hat{\delta}_i(t)$ is the estimate of $\delta_i(t)$. Note that the algorithm (52) is equivalent to the AA-DAT algorithm (24) in Section IV when we denote $x_i(t) = p_i(t) - b_i$.

Assume that the injected false data $\delta_i(t)$ is set as in (50) and is extended to a two-dimensional vector. The parameters $\alpha, \gamma, \beta_1, \beta_2$ are the same as in Section V-B. By implementing the proposed AA-DAT algorithm (52) and the extended state observer (25), the motion trajectories of the UGVs and the UAVs are shown in Fig. 13 and the tracking error is shown in Fig. 14. We can see that the objective of the distributed formation control of UAVs is successfully achieved in the presence of FDIAs by using the proposed AA-DAT algorithm.

VI. Conclusion

In this paper, we mainly focused on the study of DAT algorithms for networked control systems in the presence of external disturbances and FDIAs. For the external disturbances with known dynamics and the FDIAs with unknown dynamics, we proposed two extended DAT algorithms with a stand-alone disturbance observer and an extended state observer based on the ideas of DOBC and ADRC, respectively. The proposed two algorithms are able to eliminate the impacts of external disturbances and FDIAs while achieving the accurate tracking objective with zero steady-state error asymptotically. In the future, we will extend the results to more general scenarios in the presence of external disturbances and FDIAs.

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