Nonlinear Dyn
DOI 10.1007/s11071-015-2267-y

ORIGINAL PAPER

Event-triggered control for multi-agent network with limited digital communication

Lan Gao · Xiaofeng Liao · Huaqing Li · Guo Chen

Received: 14 January 2015 / Accepted: 7 July 2015

Abstract This paper considers the consensus problem of digital multi-agent networks by employing dynamic encode/decode technology and distributed event-triggered control strategy. In this paper, a novel integrated communication framework employing dynamic encode/decode technology and event-triggered control strategy is designed to describe the communication process in digital channels. According to this framework, a specific communication algorithm is provided in integrated communication environment. A distributed event-triggered condition that only depends on the local information of network agents is developed, and the relevant consensus analysis is given to explain the sufficiency and reasonability. Furthermore, a one-bit quantized scheme is proposed concomitantly, from which it is shown that the global consensus can still be reached though the number of bits that are responsible for information exchange between agents at each quantized transmission is only one bit. Finally, simulation results are given to verify the effectiveness of proposed approach and the correctness of theoretical results.

Keywords Distributed event triggering · Multi-agent network · Limited communication · One-bit quantization

L. Gao (✉)
The College of Computer Science, Chongqing University, Chongqing 400044, China
e-mail: gaoblan@126.com

X. Liao · H. Li
The College of Electronics and Information Engineering, Southwest University, Chongqing 400715, China
e-mail: xfliao@cqu.edu.cn

H. Li
e-mail: huaqingli@hotmail.com

G. Chen
School of Electrical and Information Engineering, The University of Sydney, Sydney 2006, Australia
e-mail: andyguochen@gmail.com

1 Introduction

The consensus and cooperation problems of multi-agent network have received increasing attention in recent years from various fields such as consensus algorithms [1,2], cooperative control of autonomous robots [3,4], flocking of unmanned air vehicles [5], rendezvous of autonomous vehicles [6,7] and so on. In fact, the digital devices such as A/D and D/A converters, discrete-level actuators/sensors, and communication channels are often embedded in multi-agent networks, so the digital communication technology is indispensable to make the network system more robust and low cost.

In communication network equipped with digital sensors and actuators, the quantized technology has become one of the major topics because the precise state information between sensors cannot be exchanged under the constraint that the analog signals have to be quantized and encoded by a finite number of bits before

Published online: 18 July 2015

being transmitted through a digital communication channel [11]. Kashhap firstly investigated the quantization communication based on integer-valued quantization in [14,15], and the extended real-valued quantization scheme can be found in [16,17]. Since the distributed averaging algorithms fail to achieve the strictly true consensus when the deterministic static uniform quantizers [14–17] are adopted, then the dynamic quantization algorithm is developed in [18]. Thereafter, the stochastic approximation methods are developed [19,20], and further, a dynamic encoding and decoding scheme with finite-level quantization is proposed in [21], in which the average consensus can be achieved based on merely one-bit information exchange between each pair of adjacent agents at each time step.

In cooperative multi-agent networks, the study of consensus problems mainly focuses on analyzing how globally consensus behavior emerges as a result of local information interactions among individuals since the agents only share information with their neighbors locally [8–10]. Generally, each agent is equipped with a small and capability-limited embedded microprocessor, which is responsible for collecting information and actuating controller updates according to some rules. So an important challenge in multi-agent systems is to design and implement decentralized algorithms for control and communication of agents.

To reduce controller updates and communication load, event-triggered control offers a new point of view on how information could be sampled and transmitted. In multi-agent systems, an agent transmits its local state information to its neighbors only when it is necessary, that is, only when a measurement of the local agent state error reaches a specified threshold [12,13]. Tabuada [12] creatively presented a triggering condition based on norms of the state and the state error $e = x(t_k) - x(t)$, that is, the last measured state minus the current state of the agent, where the measurement received at the controller is held constant until a new measurement arrives. When this happens, the error is set to zero and starts increasing until it triggers a new measurement update. Obviously, the real-time updates of controllers are avoided, and the communication load is reduced largely. The recent years have witnessed a growing interest in event-triggered strategy for coordination and cooperative control of multi-agent networks, and the specific works refer to [23–28].

The advantage of event-triggered mechanism drives technology integration and innovation that make event-triggered control applicable to digital systems and control field. The technology combination of event-triggered strategy and quantization scheme can be found in [29–32], and the time-delay relevant effects are also investigated in [33–35]. Even though these prior works have provided fundamental conclusion, it has been found that the studied quantizers are simple uniform or dynamic quantization schemes which have to occupy higher bandwidth to estimate and transmit state information. How to make full use of the limited precious bandwidth in digital communication channel is the best challenge in this paper. Inspired by the previous works [21,22], we further introduce the event-triggered mechanism into digital communication networks.

The main contribution of this paper mainly involves four points. First, we designed a novel integrated communication framework for digital multi-agent network, in which the event-triggered strategy and dynamic encode/decode scheme play an important role in communication process. Second, a distributed triggering condition that only depends on local state information of neighbor agents is developed, and the corresponding consensus analysis is provided. Third, we gave the specific communication algorithm considering dynamic encode/decode scheme under event-triggered strategy, and to design a proper quantization factor, we also proposed a self-adaptive quantization algorithm that builds a connection between quantization level and quantization factor. Last, we proposed an improved communication strategy named one-bit quantized scheme such that the global consensus can still be achieved based on only one-bit information exchange between agents at each quantized transmission. In a word, the communication load of whole network is reduced to the largest extent by utilizing the proposed integrated communication scheme.

The remainder of this paper is organized as follows: Sect. 2 declares some preliminary knowledge about graph theory and encode/decode technology; Sect. 3 provides the specific communication details for the digital network equipped with encoders/decoders under event-triggered strategy; Sect. 4 gives the consensus analysis for digital multi-agent network under the proposed event-triggered condition. Section 5 provides a one-bit quantized scheme; some numerical simulations are given in Sect. 6 to verify the main results; finally, the paper is concluded in Sect. 7.

2 Preliminaries

2.1 Notations

The following standard notations are used throughout this paper. The set of all natural number, positive integer and real number are, respectively, denoted by $\mathbb{N}$, $\mathbb{N}^+$ and $\mathbb{R}$. For a given positive number x, the maximum integer less than or equal to x is denoted by $\lfloor x \rfloor$, and the minimum integer greater than or equal to x is denoted by $\lceil x \rceil$. The absolute value of real number y is denoted by $|y|$. Let $\mathbf{1}_N$ and $\mathbf{0}_N$ be a 1 vector and a 0 vector containing N elements, respectively, and I_N be a N dimension unity matrix. The transposes of a vector v and a matrix M are denoted by v^T and M^T, respectively.

2.2 Algebraic graph theory

Let $\mathscr{G} = \{\mathscr{V}, \mathscr{E}, W\}$ be an undirected graph with N nodes, in which $\mathscr{V} = \{1, 2, ..., N\}$ is the node set, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the edge set and $W = (w_{ij}) \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix of $\mathscr{G}$. An edge $e_{ji} = (v_j, v_i)$ represents that node j can reach node i or node i can directly receive information from node j. Note that W is a symmetric matrix and $e_{ji} = e_{ij}$, i.e., the communication channels between network nodes are two way. If there is a communication channel between node i and node j, then these two nodes are called neighbors of each other and $w_{ij} = w_{ji} > 0$; otherwise, $w_{ij} = w_{ji} = 0$. The neighbor node set of node i is denoted by $\mathscr{N}_i$, while we indicate with $N_i = |\mathscr{N}_i|$ the number of neighbors of node i. The Laplacian matrix $L = (l_{ij}) \in \mathbb{R}^{N\times N}$ associated with the adjacency matrix W is defined by $l_{ij} = -w_{ij}, i \neq j$, $l_{ii} = \sum_{j=1, j\neq i}^{N} w_{ij}$ which ensures that $\sum_{j=1}^{N} l_{ij} = 0$. Generally speaking, the Laplacian matrix of an undirected graph is symmetric, and the Laplacian matrix of a directed graph is asymmetric.

2.3 Dynamic encode/decode scheme for digital channel

Digital communication technology becomes a widely used standard in modern communication system since digital signal has incomparable advantages on robustness and security compared with analog signal. Each digital channel is equipped with a pair of encoder and decoder which are responsible for encoding and decoding transmission information. As is well known, each transmission message can only be symbol data in a finite or countable set that is designed in advance. Here we assume the digital channel is reliable and the transmitted symbol data can be received without error. The encoder Φ_i of agent i for digital channel is defined as follows [21]:

$$\begin{cases} \xi_i(0) = 0, \\ \xi_i(t) = \eta_i(t)\phi_i(t) + \xi_i(t-1), \\ \phi_i(t) = q\left(\frac{1}{\eta_i(t)}(x_i(t) - \xi_i(t-1))\right), \quad t = 1, 2, \ldots, \end{cases} \tag{1}$$

where $x_i(t)$ is the precise real-value state that will be transmitted through digital channel, $\xi_i(t)$ is the internal state of encoder Φ_i, and ϕ_i is the output of Φ_i. Note we here employ a dynamic finite-level uniform quantizer $q(\cdot)$ which can transform a real value to a symbolic data, and $\eta_i(t)$ is a scaling function which is also called dynamic quantization factor.

Define the quantizer $q(\cdot)$ as a map: $\mathbb{R} \to S$, where $S = \{0, \pm i\Delta, i = 1, 2, \ldots, K\}$ is the prescribed output set and the number of quantization level is $2K + 1$. The detailed definition of quantizer $q(\cdot)$ is given by:

$$q(x) = \begin{cases} 0, & -\frac{1}{2}\Delta < x < \frac{1}{2}\Delta, \\ n\Delta, & \frac{2n-1}{2}\Delta \leq x < \frac{2n+1}{2}\Delta, n = 1, 2, \ldots, K, \\ K\Delta, & x \geq \frac{2K+1}{2}\Delta, \\ -q(-x), & x \leq -\frac{1}{2}\Delta, \end{cases} \tag{2}$$

where Δ is the quantization interval and $M = K\Delta$ is called the saturate value. In real network, the communication device always only provides the limited voltage value, which leads to a limited saturate value. Once the quantization value exceeds the saturate value, the quantizer cannot work very well owing to the unreasonable quantization error. Thus, we need to design a scheme to avoid the occurrence of this condition in the later section.

When the neighbor agents receive the symbolic data from agent i, the data are firstly decoded by corresponding decoder Ψ_i which is defined as [21]:

$$\begin{cases} \varphi_i(0) = 0, \\ \varphi_i(t) = \eta_i(t)\phi_i(t) + \varphi_i(t-1), \end{cases} \tag{3}$$

where $\varphi_i(t)$ is the output of decoder Ψ_i, i.e., the state estimate of $x_i(t)$ received by neighbors is $\varphi_i(t)$. Note from definitions (1) and (3), we can get $\varphi_i(t) = \xi_i(t)$.

3 Problem statement and protocol design

With the development of consensus and cooperation problems, lots of research works have focused on the effects of digital communication network [14–21]. In real digital network, the precise state information between agents is not available, i.e., the information exchanges through agent network are all state estimates. The relevant works have been studied, and the usual protocol is the following:

$$x_i(t+1) = x_i(t) + hu_i(t), \quad t = 0, 1, 2, \ldots, \tag{4}$$

where $x_i(t) \in \mathbb{R}$ is the state of agent i, $h > 0$ is the step size, and the controller of agent i is defined in detail as

$$u_i(t) = \sum_{j=1}^{N} w_{ij}(\hat{x}_j(t) - \hat{x}_i(t)),$$

where $\hat{x}_i(t)$ and $\hat{x}_j(t)$ are the state estimates of agent i and its neighbors, respectively.

In the following, we consider the event-triggered control strategy for multi-agent network. Assume the sequence of event times for each agent i is $0 = t_0^i, t_1^i, t_2^i, \ldots$, and the agent broadcasts its state only at its event time. Then the continuous measurements from neighbors are not available for each agent i; thus, we design the controller by utilizing the last measurements received from each neighbor $j \in \mathcal{N}_i$ as follows:

$$u_i(t) = \sum_{j=1}^{N} w_{ij}\left(\hat{x}_j\left(t_{k_j}^j\right) - \hat{x}_i\left(t_{k_i}^i\right)\right), \quad t \in [t_{k_i}^i, t_{k_i+1}^i), \tag{5}$$

where $t_{k_i}^i = \arg\min_{k_i \in \mathbb{N}, t > t_{k_i}^i}\{t - t_{k_i}^i\}$, and $\hat{x}_i(t_{k_i}^i)$, $\hat{x}_j(t_{k_j}^j)$ represent the estimates of last measurement state from agent i and its neighbors, respectively.

As a result, each agent i executes triggering only at its individual event times $t_{k_i}^i$ and then transmits its quantized state to all its neighbors. Meanwhile, the agent i updates its controller by utilizing its own measurement state estimate and its neighbors' measurement state estimate only when agent i triggers or its neighbors trigger. That is to say, in time interval $[t_{k_i}^i, t_{k_i+1}^i)$, controller of each agent i will remain unchanged as a constant $u_i(t_{k_i}^i, t_{k_j}^j)$ until its next triggering time instant $t_{k_i+1}^i$ comes or there exists at least one neighbor agent triggers.

Combined with the above encode/decode scheme, a novel integrated communication framework is designed as Fig. 1. According to this framework, we here provide a formal and detailed algorithm to describe the communication process through a dynamic encode/decode scheme and event-triggered strategy. Assume each agent i has a memory that can store its own instant state $x_i(t)$, state estimate $\hat{x}_i(t)$ and its all neighbor state estimates $\hat{x}_j(t)$, $j \in \mathcal{N}_i$. Furthermore, the initial states of all agents are given as $x(0) = (x_1(0), \ldots, x_N(0))^T$, and the all initial event time t_0^i and all state estimates $\hat{x}_i(0), i = 1, 2, \ldots, N$ are initialized to 0. Then each agent should implement the following algorithm at each time instant.

Remark 1 In this framework, the events play a key role in communication process. Only when the event is triggered, the agent's encoder begins to work and

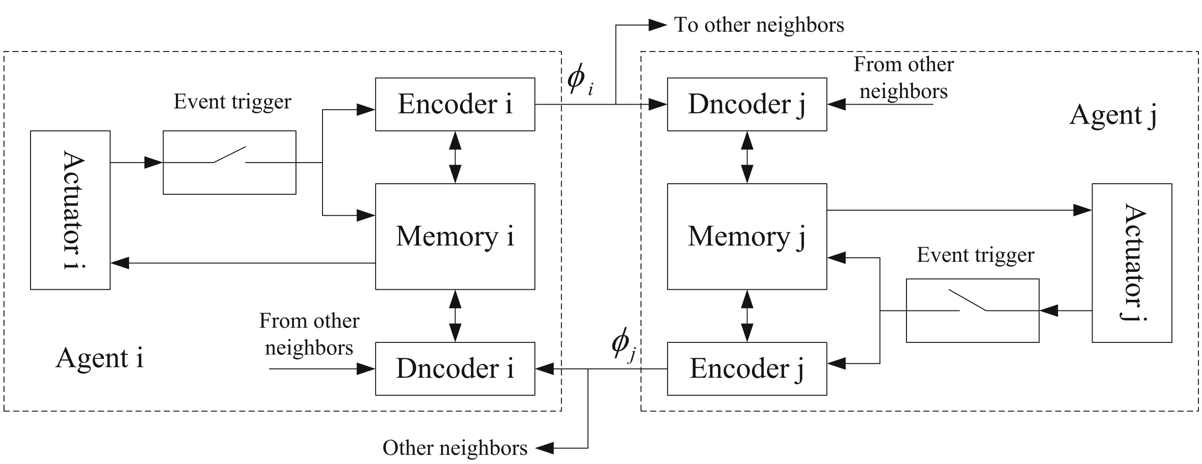

Fig. 1 The integrated communication framework

Algorithm 1 The description of communication process

```
The agent i updates its own state according to protocol x_i(t) = x_i(t-1) + h \sum_{j=1}^{N} w_{ij}(\hat{x}_j(t^j_{k_j}) - \hat{x}_i(t^i_{k_i}))
Detect the output from neighbor agents
if receive the output \phi_j(t) from neighbor j then
    updates the local memory of state estimate associated with neighbor j as follows: \hat{x}_j(t^j_{k_j+1}) = \hat{x}_j(t^j_{k_j}) + \eta_j(t)\phi_j(t)
else
    keep the local memory constant
end if
Judge whether the event happens or not
if event triggers then
    agent i broadcasts to all its neighbors the symbolic data \phi_i(t) = q(\frac{1}{\eta_i(t)}(x_i(t) - \hat{x}_i(t^i_{k_i}))), and updates its own state estimate \hat{x}_i(t^i_{k_i+1}) = \hat{x}_i(t^i_{k_i}) + \eta_i(t)\phi_i(t)
else
    keep silent
end if
```

transmits the quantization result to neighbor agents. Correspondingly, the neighbors' decoders receive the symbolic data and decode this data to obtain the state estimates. In other words, the work time of encoder/decoder is the event time of relevant agents rather than all time instants. Therefore, the operated objects of encoder/decoder are the corresponding measurement states and measurement state estimates as in Algorithm 1.

4 Consensus analysis under encode/decode scheme and event-triggered mechanism

In the multi-agent network with digital communication channels, we usually employ a variable called estimate error to describe the degree that the state estimate deviates from the corresponding precise state. Now with the presence of event-triggered strategy, we need to define a new variable named measurement error as follows:

$$\hat{e}_i(t) = \hat{x}_i\left(t^i_{k_i}\right) - x_i(t), \quad t \in [t^i_{k_i}, t^i_{k_i+1}), i = 1, 2, \ldots, N. \tag{6}$$

It represents the degree that the present time precise state deviates from the last sample time state estimate. When the measurement error reaches a threshold prescribed in advance, the event is triggered and then the agent begins to broadcast its state information.

Let $A = (a_{ij})$ with $a_{ij} = hw_{ij} \geq 0$ for $i \neq j$ and $a_{ii} = 1 - \sum_{j=1, j\neq i}^{N} a_{ij}$. Note here we assume $a_{ii} > 0$, then A is said to be double stochastic because A satisfies $A\mathbf{1} = \mathbf{1}$ and $A = A^T$. From (5) and (6), the protocol (4) can be rewritten as

$$\begin{aligned} x_i(t+1) &= x_i(t) + h\sum_{j=1}^{N} w_{ij}\left(\hat{x}_j\left(t^j_{k_j}\right) - \hat{x}_i\left(t^i_{k_i}\right)\right) \\ &= x_i(t) + \sum_{j\in\mathcal{N}_i} a_{ij}\left(\hat{x}_j\left(t^j_{k_j}\right) - \hat{x}_i\left(t^i_{k_i}\right)\right) \\ &= x_i(t) + \sum_{j=1}^{N} a_{ij}\hat{x}_j\left(t^j_{k_j}\right) - \hat{x}_i\left(t^i_{k_i}\right) \\ &= -\hat{e}_i(t) + \sum_{j=1}^{N} a_{ij}\hat{x}_j\left(t^j_{k_j}\right). \end{aligned} \tag{7}$$

Considering that each agent can only obtain its neighbors' symbolic data, the event is also computed only depending on local information that is available to each agent. We propose the following event-triggering condition:

$$\hat{e}_i^2(t) \geq \frac{a_{ii}^2}{4}\sum_{j\in\mathcal{N}_i} a_{ij}\left(\hat{x}_j\left(t^j_{k_j}\right) - \hat{x}_i\left(t^i_{k_i}\right)\right)^2, \tag{8}$$

where $t \in [t^i_{k_i}, t^i_{k_i+1})$. Once the condition (8) is satisfied, the event is triggered.

Remark 2 Note that the above event-triggering condition is truly distributed. In this design, not only the required states information is local but also the key parameters are also local. Therefore, the above developed event-triggering threshold has an obvious advantage under real condition where the global information of communication graph is not available.

Theorem 1 *Consider multi-agent network (4) with control input (5) under event-triggering condition (8). Assume that the communication graph is undirected and connected. Then the consensus can be reached, and all agents asymptotically converge to their initial state average, i.e.,* $\{x_c \in \mathbb{R} | \Delta V(t) = 0\} = \frac{1}{N}\sum_{i=1}^{N} x_i(0)$.

Proof We here firstly discuss the objective consensus states. Substituting (5) and (6) into protocol (4), we can obtain the following compact matrix form:

$$x(t+1) = (1 - hL)x(t) - hL\hat{e}(t), \tag{9}$$

where $x(t) = (x_1(t), \ldots, x_N(t))^T$, and $\hat{e}(t) = (\hat{e}_1(t), \ldots, \hat{e}_N(t))^T$.

Let $J_N = \left(\frac{1}{N}\right)\mathbf{1}\mathbf{1}^T \in \mathbb{R}^{N\times N}$, since $J_N L = \mathbf{0}$; then from (9), we can have the equality

$$\frac{1}{N}\sum_{i=1}^{N} x_i(t+1) = \frac{1}{N}\sum_{i=1}^{N} x_i(t), \quad t = 0, 1, \ldots, \tag{10}$$

which implies the state average remains constant as time goes on, i.e., $\frac{1}{N}\sum_{i=1}^{N} x_i(t) \equiv \frac{1}{N}\sum_{i=1}^{N} x_i(0)$.

For the proof of convergence under event-triggered condition (8), please see "Appendix." □

5 One-bit quantized scheme

Since the supporting quantization value is often limited by the voltage value of real communication device, thus the dynamic quantization factor in encoder (1) plays an important role in avoiding saturate quantization. In this section, we will provide an effective quantized scheme to select the proper quantization factor and to save the precious communication bandwidth to the largest extent.

Note the quantized content $x_i(t) - \xi_i(t-1) \to 0$ as $t \to \infty$ when the consensus is achieved asymptotically. According to this character, in order to precisely quantize the real-value state information, we should design a proper quantization factor including the following two properties:

(i) the function $\eta_i(t)$ should decrease as time goes on;
(ii) the function $\eta_i(t)$ should be large enough to make the saturate quantization does not happen.

Assume the quantization level is $2K + 1$, and the sequence of quantization factor $\eta_i(t)$ is denoted by $\eta_0^i, \eta_1^i, \ldots, \eta_{m-1}^i, \eta_m^i, \ldots$. The initial states of all agents are given as $x(0) = (x_1(0), \ldots, x_N(0))^T$ and $\max_i |x_i(0)| \le C_0$. Then we design the following self-adaptive avoidance algorithm for quantizer:

Algorithm 2 The self-adaptive avoidance algorithm

1: $\eta_0^i = \frac{C_0}{K\Delta}$ ▷ Initialize $\eta_i(t)$
2: **if** encoder Φ_i received the real-value state $x_i(t_{k_i}^i)$ at event time instant $t_{k_i}^i$ **then**
3: $m = m + 1$
4: $\eta_m^i = \gamma \eta_{m-1}^i, \quad 0 < \gamma < 1$
5: **while** $q\left(\frac{1}{\eta_m^i}(x_i(t_{k_i}^i) - \hat{x}_i(t_{k_i-1}^i))\right) > K\Delta$ **do**
6: $\eta_m^i = (1/\gamma)\eta_m^i$
7: **end while**
8: **end if**

According to the above Algorithm 2, we can select a proper quantization factor for any given quantization

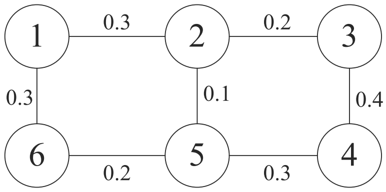

Fig. 2 The weighted interaction network with six agents

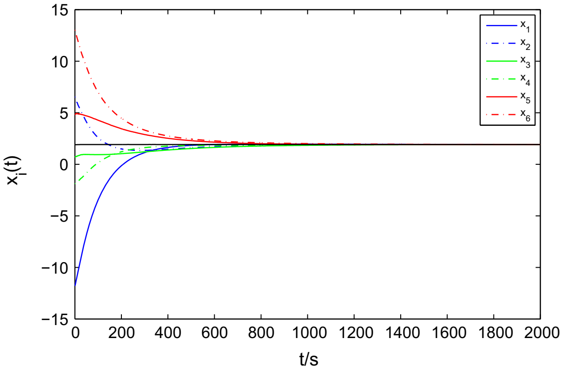

Fig. 3 The position evolution of agent i, $i = 1, 2, \ldots, 6$

level $2K + 1$ with $K \in \mathbb{N}^+$. Note that if the quantization level of quantizer is $2K + 1$, then the required bit number of each data transmitting is $\lfloor \log_2(2K) \rfloor + 1$. In the view of saving communication bandwidth, one will wonder how many bits are necessary for each information transmitting between agents. Naturally, when $K = 1$, we then get the least three-level quantizer

$$q(x) = \begin{cases} 0, & -\frac{1}{2} < x < \frac{1}{2}, \\ 1, & x \ge \frac{1}{2}, \\ -1, & x \le -\frac{1}{2}, \end{cases} \tag{11}$$

in which condition, the necessary bit number is two.

Remark 3 We can further reduce the number of necessary bits to only one by carrying out the following improved communication strategy: When the output result of quantizer Φ_i is zero, the quantizer does not broadcast its output to neighbor agents. From decoder (3), the last measurement state estimate is equal to the previous measurement state estimate when the received symbolic data are zero. Thus the improved communication strategy is reasonable and a $2K+1$-level quantizer requires at least $\lceil \log_2(2K) \rceil$ bits to transmit quantized data without error under this improved strategy. Especially, the three-level quantizer only needs one bit to send its output result.

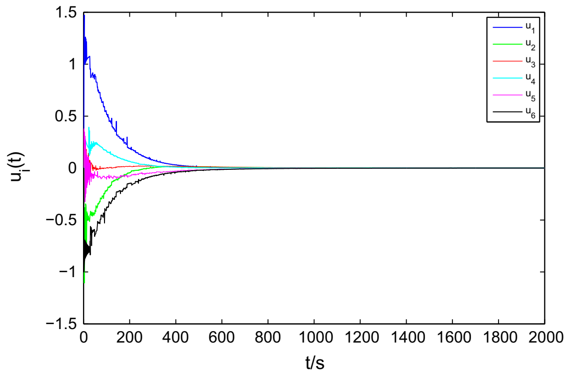

Fig. 4 The velocity evolution of agent i, $i = 1, 2, \ldots, 6$

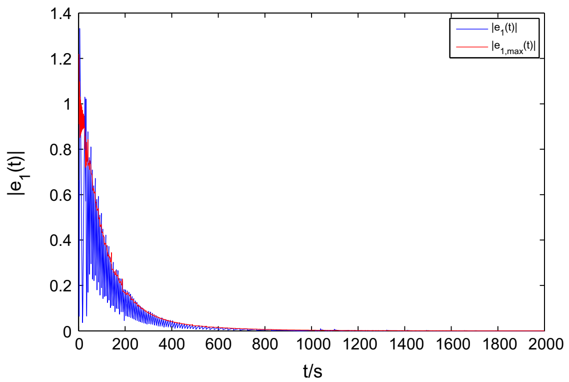

Fig. 5 The evolution of measurement error and threshold

6 Simulations

In this section, we provide some simulations to verify and illustrate the proposed framework and the one-bit quantized scheme. Consider the information interactive network with communication graph $\mathscr{G}$ given in Fig. 2, and the initial state values of agents are randomly generated in the interval $[-20, 20]$. To save communication bandwidth, we employ a one-bit quantizer, i.e., each information transmitting only uses one bit to exchange symbolic data. We set quantization interval $\Delta = 1$, and $C_0 = 20$, $\gamma = 0.9$, and then a group of simulation results are obtained.

Figure 3 shows the state evolutions of all agents, in which all agents' state average denoted by black line holds constant and the objective consensus state converges to this state asymptotically. Figure 4 shows the controller state evolutions of all agents. The evolution of measurement error of first agent is shown in Fig. 5, in which $|e_1(t)| = |\hat{e}_1(t)|$ is the measurement error of agent 1 and $|e_{1,max}(t)| = \sqrt{\frac{a_{ii}^2}{4}\sum_{j\in\mathscr{N}_i} a_{ij}\left(\hat{x}_j(t_{k_j}^j) - \hat{x}_i(t_{k_i}^i)\right)^2}$ is the specified maximum threshold. Note the measurement error cannot be reset to zero owing to the existence of quantization error in encode/decode process. In Fig. 6, the events of each agent are marked in time interval $[0, 200]$, from which we can see that the sampling is sporadic rather than every time instant. To show the event interval time more clearly, we plot the event step size in Fig. 7, from which we can easily see that the event step size is not homogeneous.

7 Conclusion

In this paper, we have studied the consensus problem for digital multi-agent networks equipped with encoders/decoders under event-triggered control strategy. We not only have proposed a integrated communication framework considering dynamic encode/decode technology and event-triggered strategy, but also have given the specific communication algorithm and derived the sufficient condition for reaching global consensus. Finally, we have also provided a one-bit quantized scheme which shows that the global consensus of digital network can still be reached based on only one-bit information exchange between agents

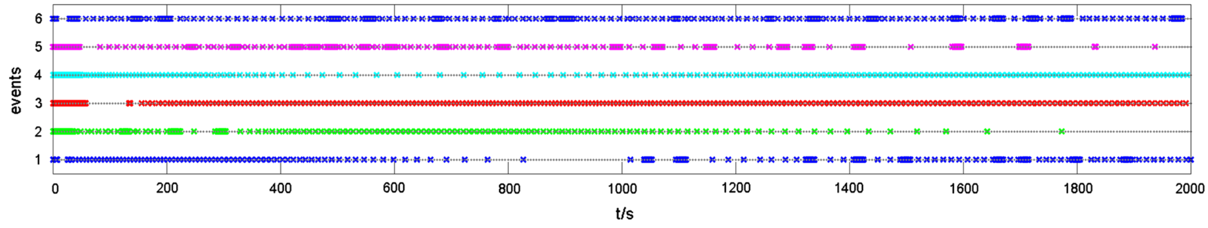

Fig. 6 Event-triggering times of agent i, $i = 1, 2, \ldots, 6$

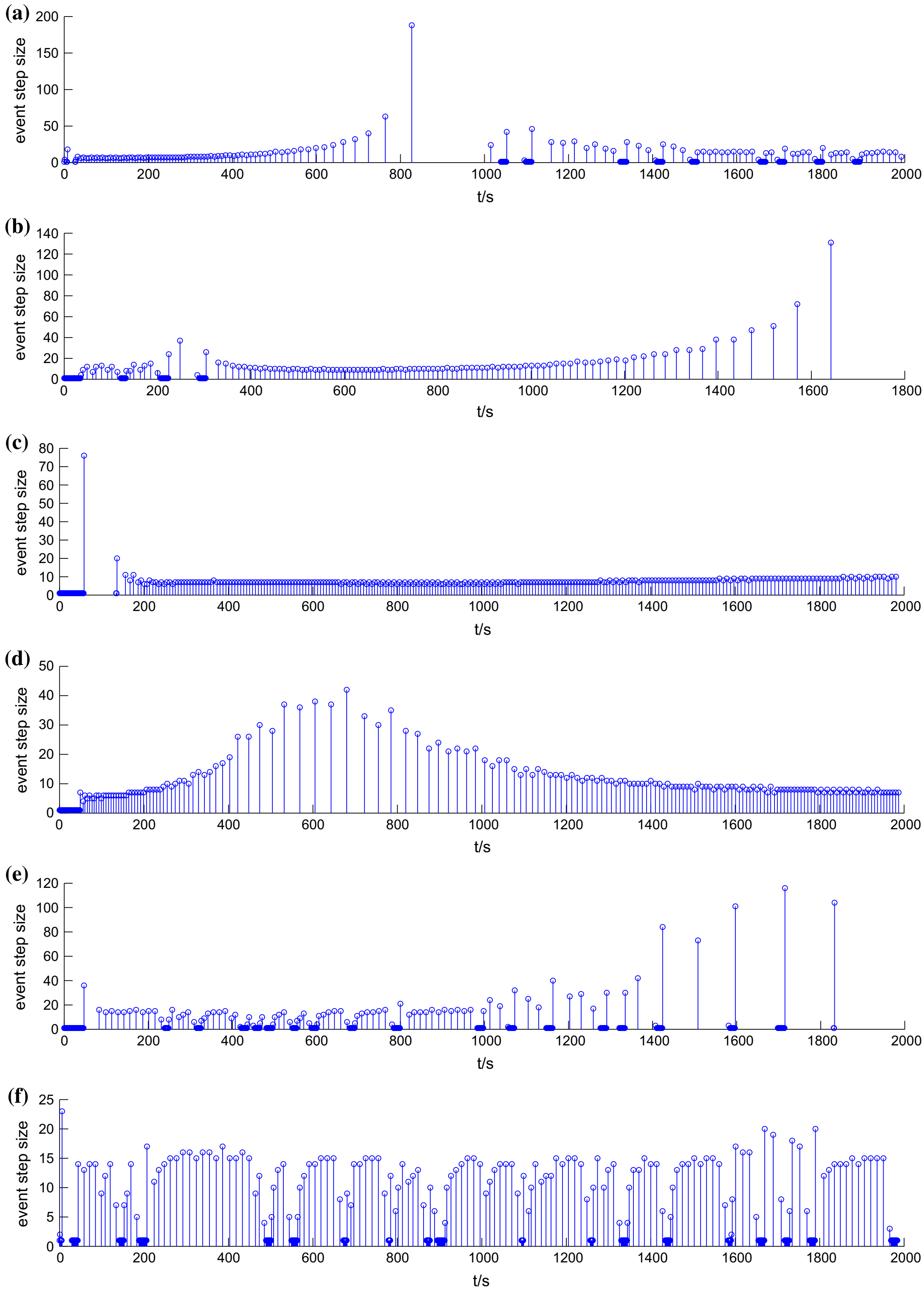

Fig. 7 Event step size of agents. **a** Agent 1, **b** Agent 2, **c** Agent 3, **d** Agent 4, **e** Agent 5, **f** Agent 6

at each quantized transmission. In the future, we will consider extending this integrated communication scheme to investigate the consensus problem of higher-order multi-agent networks with directed or switching topologies.

Acknowledgments This work was supported in part by the National Natural Science Foundation of China under Grant 61472331, in part by the Research Fund of Preferential Development Domain for the Doctoral Program of Ministry of Education of China under Grant 201101911130005, in part by the State Key Laboratory of Power Transmission Equipment and System Security and New Technology, Chongqing University, under Grant 2007DA10512709207, in part by the Fundamental Research Funds for the Central Universities under Grant XDJK2014C117, in part by the Natural Science Foundation Project of Chongqing CSTC under Grant cstc2014jcyjA40016 and in part by the Natural Science Foundation of China under Grant 61403314.

Appendix: Proof of Theorem 1

Proof Consider the following Lyapunov functional candidate

$$V(t)=\sum_{i=1}^{N}x_i^2(t), \tag{12}$$

then

$$\begin{aligned}\Delta V(t)&=V(t+1)-V(t)\\&=\sum_{i=1}^{N}x_i^2(t+1)-\sum_{i=1}^{N}x_i^2(t)\\&=\sum_{i=1}^{N}\left[\hat{e}_i^2(t)-2\sum_{j=1}^{N}a_{ij}\hat{e}_i(t)\hat{x}_j\left(t_{k_j}^j\right)\right.\\&\quad\left.+\left(\sum_{j=1}^{N}a_{ij}\hat{x}_j\left(t_{k_j}^j\right)\right)^2\right]-\sum_{i=1}^{N}x_i^2(t).\end{aligned} \tag{13}$$

Since

$$\begin{aligned}&\sum_{j=1}^{N}a_{ij}\hat{e}_i(t)\hat{x}_j\left(t_{k_j}^j\right)\\&\quad=\sum_{j=1,j\neq i}^{N}a_{ij}\hat{e}_i(t)\hat{x}_j\left(t_{k_j}^j\right)+a_{ii}\hat{e}_i(t)\hat{x}_i\left(t_{k_i}^i\right)\\&\quad=\sum_{j=1,j\neq i}^{N}a_{ij}\hat{e}_i(t)\hat{x}_j\left(t_{k_j}^j\right)\\&\qquad+\left(1-\sum_{j=1,j\neq i}^{N}a_{ij}\right)\hat{e}_i(t)\hat{x}_i\left(t_{k_i}^i\right)\\&\quad=\sum_{j=1,j\neq i}^{N}a_{ij}\hat{e}_i(t)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)\\&\qquad+\hat{e}_i(t)\hat{x}_i\left(t_{k_i}^i\right),\end{aligned}$$

$$\begin{aligned}&\left(\sum_{j=1}^{N}a_{ij}\hat{x}_j\left(t_{k_j}^j\right)\right)^2\\&\quad=\sum_{j=1}^{N}a_{ij}^2\hat{x}_j^2\left(t_{k_j}^j\right)+2\sum_{j=1}^{N}\sum_{l>j}^{N}a_{ij}a_{il}\hat{x}_j\left(t_{k_j}^j\right)\hat{x}_l\left(t_{k_l}^l\right),\end{aligned}$$

we have

$$\begin{aligned}&\Delta V(t)\\&=\sum_{i=1}^{N}\left(\sum_{j=1}^{N}a_{ij}^2\hat{x}_j^2\left(t_{k_j}^j\right)+2\sum_{j=1}^{N}\sum_{l>j}^{N}a_{ij}a_{il}\hat{x}_j\left(t_{k_j}^j\right)\hat{x}_l\left(t_{k_l}^l\right)\right)\\&\quad-2\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}\hat{e}_i(t)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)\\&\quad+\sum_{i=1}^{N}\left(\hat{e}_i^2(t)-x_i^2(t)-2\hat{e}_i(t)\hat{x}_i\left(t_{k_i}^i\right)\right)\\&=\sum_{i=1}^{N}\left(\sum_{j=1}^{N}a_{ij}^2\hat{x}_j^2\left(t_{k_j}^j\right)+2\sum_{j=1}^{N}\sum_{l>j}^{N}a_{ij}a_{il}\hat{x}_j\left(t_{k_j}^j\right)\hat{x}_l\left(t_{k_l}^l\right)\right)\\&\quad-2\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}\hat{e}_i(t)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)-\sum_{i=1}^{N}\hat{x}_i^2\left(t_{k_i}^i\right)\\&=\sum_{i=1}^{N}\left[\sum_{j=1}^{N}a_{ij}^2\hat{x}_j^2\left(t_{k_j}^j\right)+\sum_{j=1}^{N}\sum_{l>j}^{N}a_{ij}a_{il}\left(\hat{x}_j^2\left(t_{k_j}^j\right)+\hat{x}_l^2\left(t_{k_l}^l\right)\right)\right]\\&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l>j}^{N}a_{ij}a_{il}\left(-\hat{x}_j^2\left(t_{k_j}^j\right)-\hat{x}_l^2\left(t_{k_l}^l\right)\right.\\&\qquad\left.+2\hat{x}_j\left(t_{k_j}^j\right)\hat{x}_l\left(t_{k_l}^l\right)\right)-2\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}\\&\qquad\times\hat{e}_i(t)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)-\sum_{i=1}^{N}\hat{x}_i^2\left(t_{k_i}^i\right).\end{aligned} \tag{14}$$

Note that

$$\begin{aligned}&\sum_{i=1}^{N}\left[\sum_{j=1}^{N}a_{ij}^2\hat{x}_j^2\left(t_{k_j}^j\right)+\sum_{j=1}^{N}\sum_{l>j}^{N}a_{ij}a_{il}\left(\hat{x}_j^2\left(t_{k_j}^j\right)+\hat{x}_l^2\left(t_{k_l}^l\right)\right)\right]\\&=\sum_{i=1}^{N}\left[\sum_{j=1}^{N}a_{ij}^2\hat{x}_j^2\left(t_{k_j}^j\right)+\sum_{j=1}^{N}\sum_{l=1,l\neq j}^{N}a_{ij}a_{il}\hat{x}_j^2\left(t_{k_j}^j\right)\right]\end{aligned}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} a_{ij}a_{il}\hat{x}_j^2\left(t_{k_j}^j\right)$$
$$= \sum_{j=1}^{N}\hat{x}_j^2\left(t_{k_j}^j\right), \tag{15}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l>j}^{N} a_{ij}a_{il}\left(-\hat{x}_j^2\left(t_{k_j}^j\right)-\hat{x}_l^2\left(t_{k_l}^l\right)\right.$$
$$\left.+2\hat{x}_j\left(t_{k_j}^j\right)\hat{x}_l\left(t_{k_l}^l\right)\right)$$
$$= -\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l>j}^{N} a_{ij}a_{il}\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_l\left(t_{k_l}^l\right)\right)^2$$
$$= -\sum_{i=1}^{N}\left[\sum_{j=1,j\neq i}^{N}\sum_{l>j,l\neq i}^{N} a_{ij}a_{il}\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_l\left(t_{k_l}^l\right)\right)^2\right.$$
$$\left.+\sum_{j=1,j\neq i}^{N} a_{ij}a_{ii}\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2\right], \tag{16}$$

besides, according to the inequality $|xy| \leq \frac{\alpha}{2}x^2+\frac{1}{2\alpha}y^2$ for any $\alpha > 0$, then one has

$$-2\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} a_{ij}\hat{e}_i(t)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)$$
$$\leq \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2a_{ij}\left[\frac{1}{2\alpha_i}\hat{e}_i^2(t)+\frac{\alpha_i}{2}\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2\right]$$
$$= \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} a_{ij}\left[\frac{1}{\alpha_i}\hat{e}_i^2(t)+\alpha_i\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2\right]. \tag{17}$$

Combining Eqs. (14)–(17), we can obtain

$$\Delta V(t)$$
$$\leq -\sum_{i=1}^{N}\left[\sum_{j=1,j\neq i}^{N}\sum_{l>j,l\neq i}^{N} a_{ij}a_{il}\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_l\left(t_{k_l}^l\right)\right)^2\right.$$
$$\left.+\sum_{j=1,j\neq i}^{N} a_{ij}a_{ii}\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2\right]$$
$$+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} a_{ij}\left[\frac{1}{\alpha_i}\hat{e}_i^2(t)+\alpha_i\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2\right]$$
$$\leq \sum_{i=1}^{N}\left[\sum_{j=1,j\neq i}^{N}\frac{a_{ij}}{\alpha_i}\hat{e}_i^2\right.$$
$$\left.-\sum_{j=1,j\neq i}^{N} a_{ij}(a_{ii}-\alpha_i)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2\right]$$
$$\leq \sum_{i=1}^{N}\left[\frac{\hat{e}_i^2}{\alpha_i}-\sum_{j=1,j\neq i}^{N} a_{ij}(a_{ii}-\alpha_i)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2\right]. \tag{18}$$

To make $\Delta V(t) < 0$, the following sufficient condition is obtained

$$\hat{e}_i^2(t) < \sum_{j\in\mathcal{N}_i} a_{ij}\alpha_i(a_{ii}-\alpha_i)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2. \tag{19}$$

Thus the following event-triggered condition candidate is chosen

$$\hat{e}_i^2(t) \geq \sum_{j\in\mathcal{N}_i} a_{ij}\alpha_i(a_{ii}-\alpha_i)\left(\hat{x}_j\left(t_{k_j}^j\right)-\hat{x}_i\left(t_{k_i}^i\right)\right)^2. \tag{20}$$

Considering the efficiency of event triggering, we choose $\alpha_i = a_{ii}/2$ to make the right of (20) maximum, and then, we get the formal event-triggered condition (8). Therefore, under the event-triggered condition (8), we have $\Delta V(t) < 0$ which implies all agents will asymptotically converge to the set $\{x_c \in \mathbb{R}|\Delta V(t) = 0\}$ according to the LaSalle's invariance principle. The proof is completed.

References

1. Ren, W., Atkins, E.M.: Distributed multi-vehicle coordinated control via local information exchange. Int. J. Robust Nonlin. Contr. **17**(1011), 1002–1033 (2007)
2. Ren, W., Beard, R.W.: Consensus algorithms for double-integrator dynamics. Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications, pp. 77–104 (2008)
3. Fax, J.A., Murray, R.M.: Information flow and cooperative control of vehicle formations. IEEE Trans. Automat. Contr. **49**(9), 1465–1476 (2004)
4. Qu, Z., Wang, J., Hull, R.A.: Cooperative control of dynamical systems with application to autonomous vehicles. IEEE Trans. Automat. Contr. **53**(4), 894–911 (2008)
5. Tanner, H.G., Jadbabaie, A., Pappas, G.J.: Flocking in fixed and switching networks. IEEE Trans. Automat. Contr. **52**(5), 863–868 (2007)
6. Lin, J., Morse, A.S., Anderson, B.D.O.: The multi-agent rendezvous problem–part 1 the synchronous case. SIAM J. Contr. Optim. **46**(6), 2096–2119 (2007)
7. Lin, J., Morse, A.S., Anderson, B.D.O.: The multi-agent rendezvous problem–part 2 the asynchronous case. SIAM J. Contr. Optim. **46**(6), 2120–2147 (2007)

8. Olfati-Saber, R., Fax, J.A., Murray, R.M.: Consensus and cooperation in networked multi-agent systems. Proc. IEEE **95**(1), 215–233 (2007)
9. Li, Huaqing, Liao, Xiaofeng, Dong, Tao, Xiao, Li: Second-order consensus seeking in directed networks of multi-agent dynamical systems via generalized linear local interaction protocols. Nonlinear Dyn. **70**(3), 2213–2226 (2012)
10. Chen, Kairui, Wang, Junwei, Zhang, Yun, Liu, Zhi: Second-order consensus of nonlinear multi-agent systems with restricted switching topology and time delay. Nonlinear Dyn. **78**(2), 881–887 (2014)
11. Wen, G., Duan, Z., Yu, W., Chen, G.: Consensus in multi-agent systems with communication constraints. Int. J. Robust Nonlinear Contr. **22**(2), 170–182 (2012)
12. Tabuada, P.: Event-triggered real-time scheduling of stabilizing control tasks. IEEE Trans. Automat. Control **52**(9), 1680–1685 (2007)
13. Wang, X., Lemmon, M.D.: Event-triggered broadcasting across distributed networked control systems. In Proceedings of American Control Conference pp. 3139–3144 (2008)
14. Kashyap, A., Basar, T., Srikant, R.: Consensus with quantized information updates. In: Decision and Control, 2006 45th IEEE Conference on, pp. 2728–2733. IEEE (2006)
15. Kashyap, A., Basar, T., Srikant, R.: Quantized consensus. Automatica **43**(7), 1192–1203 (2007)
16. Carli, R., Fagnani, F., Frasca, P., Taylor, T., Zampieri, S.: Average consensus on networks with transmission noise or quantization. In: Proceedings of European Control Conference, pp. 1852–1857 (2007)
17. Frasca, P., Carli, R., Fagnani, F., Zampieri, S.: Average consensus on networks with quantized communication. Int. J. Robust Nonlinear Contr. **19**(16), 1787–1816 (2009)
18. Carli, R., Fagnani, F., Frasca, P., Zampieri, S.: Efficient quantized techniques for consensus algorithms. NeCST07, Nancy **3**, 1 (2007)
19. Huang, M., Manton, J.H.: Coordination and consensus of networked agents with noisy measurements: stochastic algorithms and asymptotic behavior. SIAM J. Contr. Optim. **48**(1), 134–161 (2009)
20. Li, T., Zhang, J.F.: Consensus conditions of multi-agent systems with time-varying topologies and stochastic communication noises. IEEE Trans. Automat. Contr. **55**(9), 2043–2057 (2010)
21. Li, T., Fu, M., Xie, L., Zhang, J.F.: Distributed consensus with limited communication data rate. IEEE Trans. Automat. Contr. **56**(2), 279–292 (2011)
22. Li, D., Liu, Q., Wang, X., Lin, Z.: Distributed consensus over directed networks with limited information communication. In: Control and Automation (ICCA), 2011 9th IEEE International Conference on, pp. 836–841. IEEE (2011)
23. Dimarogonas, D.V., Johansson, K.H.: Event-triggered control for multi-agent systems. In: Decision and Control, 28th Chinese Control Conference, 2009. Proceedings of the 48th IEEE Conference on, pp. 7131–7136. IEEE (2009)
24. Dimarogonas, D.V., Frazzoli, E., Johansson, K.H.: Distributed event-triggered control for multi-agent systems. IEEE Trans. Automat. Control **57**(5), 1291–1297 (2012)
25. Eqtami, A., Dimarogonas, D.V., Kyriakopoulos, K.J.: Event-triggered control for discrete-time systems. In: American Control Conference (ACC), 2010, pp. 4719–4724. IEEE (2010)
26. Chen, X., Hao, F.: Event-triggered average consensus control for discrete-time multi-agent systems. IET Contr. Theory Appl. **6**(16), 2493–2498 (2012)
27. Garcia, E., Cao, Y., Yu, H., Antsaklis, P., Casbeer, D.: Decentralised event-triggered cooperative control with limited communication. Int. J. Contr. **86**(9), 1479–1488 (2013)
28. Fan, Y., Feng, G., Wang, Y., Song, C.: Distributed event-triggered control of multi-agent systems with combinational measurements. Automatica **49**(2), 671–675 (2013)
29. Li, L., Wang, X., Lemmon, M.: Stabilizing bit-rates in quantized event triggered control systems. In: Proceedings of the 15th ACM International Conference on Hybrid Systems: Computation and Control, pp. 245–254. ACM (2012)
30. Zhang, Z., Zhang, L., Hao, F., Wang, L.: Distributed event-triggered consensus for multi-agent systems with quantization. Int. J. Contr. pp. 1–18 (2014)
31. Guan, Y., Han, Q.L., Peng, C.: Event-triggered quantized-data feedback control for linear systems. In: Industrial Electronics (ISIE), 2013 IEEE International Symposium on, pp. 1–6 (2013)
32. Zhang, H., Ren, Y., Wang, X.: Distributed event-triggered quantizer in multi-agent systems. J. Dyn. Syst. Meas. Contr. **136**(4), 044,504 (2014)
33. Hu, S., Yue, D.: Event-triggered control design of linear networked systems with quantizations. ISA Trans. **51**(1), 153–162 (2012)
34. Garcia, E., Antsaklis, P.J.: Model-based event-triggered control for systems with quantization and time-varying network delays. IEEE Trans. Automat. Contr. **58**(2), 422–434 (2013)
35. Li, L., Ho, D.W., Huang, C., Lu, J.: Event-triggered discrete-time multi-agent consensus with delayed quantized information. In: Control Conference (CCC), 2014 33rd Chinese, pp. 1722–1727. IEEE (2014)