Asian Journal of Control, Vol. 18, No. 4, pp. 1–9, July 2016
Published online in Wiley Online Library (wileyonlinelibrary.com) DOI: 10.1002/asjc.1166

EVENT-TRIGGERED CONTROL FOR MULTI-AGENT SYSTEMS WITH GENERAL DIRECTED TOPOLOGY AND TIME DELAYS

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ABSTRACT

Recent years have witnessed a growing interest in event-triggered strategies for coordination and cooperative control of multi-agent systems. However, the most previous works and developments focus on the interactive network that has no communication delays. This paper deals with the consensus problem of an agent system with event-triggered control strategy under communication time delays. We first propose a time delays system model, then present a novel event triggering function that not only avoids continuous communication but also excludes the Zeno behavior. Furthermore, we provide the consensus analysis using an inequality technique instead of the traditional linear matrix inequality method, and we demonstrate that the inter-event times for each agent are strictly positive, which implies that the Zeno behavior can be excluded. Finally, simulation results show the effectiveness of the proposed approach and illustrate the correctness of the theoretical results.

Key Words: Event-triggered, multi-agent systems, inequality technique, time delays.

I. INTRODUCTION

Consensus andRecent cooperation problems have a long history in multi-agent systems which consist of vast interconnected autonomous robots, vehicles or mobile sensors [1]. In cooperative multi-agent systems, the study of consensus problems mainly focuses on analyzing how global consensus behavior emerges as a result of local information interactions among individuals since the agents only share information with their neighbors locally. More recently, consensus behaviors have received increasing attention from various disciplines of engineering and science involving consensus algorithms [2–4], formation control [5,6], rendezvous [7,8], agent flocking [9] and distributed estimation [10].

An important challenge in multi-agent systems is to design and implement decentralized algorithms for control and communication of agents. Generally, each agent is equipped with a small and capability-limited embedded microprocessor, which is responsible for collecting information and actuating controller updates according to some rules. To reduce controller updates, two methods have been developed, time-scheduled and event-triggered. However, the time-scheduled method has an insurmountable deficiency that the communication and the task scheduling on control units have to be synchronized during operation in order to ensure the strict time specifications in system design [11]. Thus, the event-triggered method, as an excellent control scheme, has attracted more and more attention from many researchers of various fields.

Event-triggered control offers a new point of view on how information could be sampled and transmitted. In multi-agent systems, an agent transmits its local state to its neighbors only when it is necessary, that is, only when a measurement of the local agent state error reaches a specified threshold [12,13]. Tabuada [12] creatively presented a triggering condition based on norms of the state and the state error $e = x(t_k) - x(t)$, that is, the last measured state minus the current state of the agent, where the measurement received at the controller is held constant until a new measurement arrives. When this happens, the error is set to zero and starts increasing until it triggers a new measurement update.

Recent years also have witnessed event-triggered control mechanism improvements and developments. Event-triggered control of multi-agent systems with

Manuscript received July 18, 2014; revised November 14, 2014; accepted December 22, 2014.

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This work was supported in part by the National Natural Science Foundation of China under Grant 61170249, in part by the Fundamental Research Funds for the Central Universities under Grant XDJK2014C117, in part by the Natural Science Foundation Project of Chongqing CSTC under Grant cstc2014jcyjA40016, in part by the Natural Science Foundation of China under Grant 61403314, and in part by the Graduate Student Research Innovation Project CYB14022.

undirected topology was investigated by Dimarogonas and Johansson in [14], [15]. The limited communication for event triggering control was researched in [16], [17], in which the continuous communication of the triggering function was avoided. The pinning control considering event-triggered strategy can be found in [19] and [18]. The authors in [20] studied the distributed rendezvous problem with a distributed controller and designed the basic event-triggered algorithm, and the inequality technology was introduced for general directed topology in [21], [22]. The signal quantisation problem about event triggering control also was discussed in [16], [23].

Since the communication delays are inevitable in real information interactive systems, the time delay case of the consensus problem has attracted much investigation, including frequency-domain methods [24], linear matrix inequality (LMI) methods [25], and inequality technology [26]. To date, however, the previous works about event-triggered control of the consensus problem have all assumed the information communication is instantaneous. Therefore, in this paper, we will discuss the decentralised event-triggered control for multi-agent systems with general directed topology and time delays. Our main contribution is to present a time-delay system model and a novel event triggering function that not only truly avoids continuous communication but also excludes the Zeno behavior. Furthermore, we provide the consensus analysis using an inequality technique instead of the traditional LMI method, and we demonstrate that the inter-event times for each agent are strictly positive, which implies that the Zeno behavior can be excluded. Eventually, a group of simulation results are given to show the evolution performance by applying the proposed control strategy.

The remainder of this paper is organised as follows. Section II declares some preliminary knowledge about graph theory and the consensus problem. Section III provides the specific technical details for consensus analysis and the demonstration for absence of Zeno behavior. Section IV gives some numerical simulations to verify the main results. Finally, the paper is concluded in Section V.

II. PRELIMINARIES AND PROBLEM STATEMENT

2.1 Graph theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, A\}$ be a directed graph with N nodes, in which $\mathcal{V} = \{1, 2, \ldots, N\}$ is the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and $A = (a_{ij})_{N\times N}$ is the adjacency matrix of $\mathcal{G}$. A directed edge $e_{ji} = (v_j, v_i)$ means that node j can reach node i or node i can receive information from node j. If there is an edge from node j to node i, then node j is called a neighbor of node i and $a_{ij} = 1$; otherwise, $a_{ij} = 0$. The neighbor node set of node i is denoted by $\mathcal{N}_i$, while we indicate with $N_i = |\mathcal{N}_i|$ the number of neighbors of node i. The Laplacian matrix $L = (l_{ij})_{N\times N}$ associated with the adjacency matrix A is defined by $l_{ij} = -a_{ij}, i \neq j$; $l_{ii} = \sum_{j=1,j\neq i}^{N} a_{ij}$, which ensures that $\sum_{j=1}^{N} l_{ij} = 0$. Note that an undirected graph can be viewed as a special case of a directed graph, where the information flow is bidirectional. Generally speaking, the Laplacian matrix of a directed graph is asymmetric, and the Laplacian matrix of a undirected graph is symmetric.

2.2 Some support lemmas

Lemma 1. [27]. Assume there exists a spanning tree in digraph $\mathcal{G}$. Then, the Laplacian matrix L associated with $\mathcal{G}$ has eigenvalue 0 with algebraic multiplicity one, and the real parts of all the other eigenvalues are positive, *i.e.*, the eigenvalues satisfy $0 = \lambda_1(L) < \mathcal{R}(\lambda_2(L)) \leq \ldots \leq \mathcal{R}(\lambda_N(L))$.

Lemma 2. Suppose L is the Laplacian matrix of digraph $\mathcal{G}$ with N nodes. Then $\bar{L} - \mathbf{1}_{N-1} \cdot \bar{l}_1^T$ is positive stable (*i.e.* all eigenvalues have positive real parts) if the digraph $\mathcal{G}$ has a directed spanning tree, where $\bar{l}_1^T$ and $\bar{L}$ are defined as

$$\bar{l}_1 = \left(l_{12}, \ldots, l_{1N}\right)^T,$$

$$\bar{L} = \begin{pmatrix} l_{22} & l_{23} & \ldots & l_{2N} \\ l_{32} & l_{33} & \ldots & l_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ l_{N2} & l_{N3} & \ldots & l_{NN} \end{pmatrix}.$$

Proof. Let

$$S = \begin{pmatrix} 1 & \mathbf{0}_{N-1} \\ \mathbf{1}_{N-1} & I_{N-1} \end{pmatrix},$$

where $\mathbf{0}_{N-1}$ and $\mathbf{1}_{N-1}$ denote a 0 vector and 1 vector, respectively, containing $N-1$ elements and where I_{N-1} is a $N-1$ dimension unity matrix. Then, we can get the similar matrix of L:

$$\hat{L} = S^{-1}LS = \begin{pmatrix} 0 & \bar{l}_1^T \\ 0 & \tilde{L} \end{pmatrix},$$

where $\tilde{L} = \bar{L} - \mathbf{1}_{N-1} \cdot \bar{l}_1^T$. Due to the properties of the similar matrix, we have $\det(\lambda I_N - L) = \det(\lambda I_N - \hat{L}) = \lambda \det(\lambda I_{N-1} - \tilde{L})$. According to Lemma 1, the matrix $\tilde{L}$ is positive stable (*i.e.* all eigenvalues have positive real parts).

Lemma 3. [21]. Suppose $A \in \mathbb{R}^{N\times N}$ is a negative stable matrix and has eigenvalue real parts $\mathcal{R}(\lambda_N) < \dots < \mathcal{R}(\lambda_1) < 0$, then there exists a constant $\rho \geq 1, \alpha > 0$ such that $\|e^{At}\| \leq \rho e^{-\alpha t}$ for all $t \geq 0$.

2.3 Consensus protocols

According to the event-triggered strategy investigated in [14,15], the following consensus protocol of the agent system is defined:

$$\dot{x}_i(t)=\sum_{j=1}^{N} a_{ij}\left(x_j\left(t^j_{k_j}\right)-x_i\left(t^i_{k_i}\right)\right),\quad t\in\left[t^i_{k_i},t^i_{k_i+1}\right). \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state of agent i, $x_i\left(t^i_{k_i}\right)$ is the last measurement state of agent i, and $x_j\left(t^j_{k_j}\right)$ represents the last measurement states received from neighbors of agent i.

In this paper, we further consider the consensus protocol with communication delays. For simplicity, we assume that all communication delays are constant and equal to τ, which can be explained as an average delay. Then, the final consensus protocol is proposed as follows:

$$\begin{cases} \dot{x}_i(t) = u_i(t), \\ x_i(\theta) = \varphi_i(\theta), \quad \theta\in[-\tau,0], \end{cases} \tag{2}$$

where φ_i corresponds to the initial states of agent i in interval $[-\tau,0]$, and the controller of agent i is defined in detail as

$$u_i(t)=\sum_{j=1}^{N} a_{ij}\left(x_j\left(t^j_{k_j}-\tau\right)-x_i\left(t^i_{k_i}-\tau\right)\right).$$

III. CONSENSUS ANALYSIS

In this section, some consensus criteria is presented to guarantee the feasibility of the control model. First, the measurement error for agent i in delay communication system is defined by:

$$e_i(t)=x_i\left(t^i_{k_i}-\tau\right)-x_i(t-\tau),\quad t\in\left[t^i_{k_i},t^i_{k_i+1}\right). \tag{3}$$

This represents the degree that the present time state deviates from the last sample time state in the communication system with a constant time delay τ. When the measurement error reaches a threshold prescribed in advance, the event is triggered and the agent begins to update its controller.

Considering that each agent can only obtain its neighbors' measurements, the event is also computed only depending on local information that is available to each agent. We propose the following event triggering function:

$$\begin{aligned} &p_i\left(t,e_i(t),x_i\left(t^i_{k_i}-\tau\right),x_j\left(t^j_{k_j}-\tau\right)\right)\\ &=\|e_i(t)\|-\frac{\beta_1}{N_i}\sum_{j\in\mathcal{N}_i}\|x_j\left(t^j_{k_j}-\tau\right)-x_i\left(t^i_{k_i}-\tau\right)\|\\ &\quad-\beta_2 e^{-\gamma t}, \end{aligned} \tag{4}$$

where $\beta_1,\beta_2,\gamma > 0$ are the performance parameters. Then, an event occurs when the following condition is violated:

$$p_i\left(t,e_i(t),x_i\left(t^i_{k_i}-\tau\right),x_j\left(t^j_{k_j}-\tau\right)\right)\leq 0. \tag{5}$$

Remark 1. Once the event is triggered, the measurement error is reset to zero since at that time instant we have $e_i\left(t^i_{k_i}\right) = x_i\left(t^i_{k_i}-\tau\right) - x_i\left(t^i_{k_i}-\tau\right) = 0$, thus Inequality (5) still is satisfied. Moreover, our event triggering function does not depend on the real-time states of neighbor agents. In previous works, although the event-triggered strategy makes agents' controllers avoid continuous communication, the cost is that each agent must collect its neighbors' real-time states to calculate a triggering function and then judge whether the event is triggered or not. In other words, the communication is still continuous from the point of view of the agents. Conversely, $p_i\left(t,e_i(t),x_i\left(t^i_{k_i}-\tau\right),x_j\left(t^j_{k_j}-\tau\right)\right)$ is only dependent on agent neighbors' measurement states rather than real-time states, which can save limited communication bandwidth effectively. The next result reveals the convergence of the system model (2) using the triggering function (4).

Definition 1. The multi-agent system (2) is said to achieve global consensus if for any initial state values such that

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0,\quad \forall i,j=1,2,\dots,N.$$

Let $\varepsilon_i(t) = x_i(t) - x_1(t), i = 2,\dots,N$, and $\varepsilon(t) = (\varepsilon_2^T(t),\dots,\varepsilon_N^T(t)), e(t) = (e_2^T(t),\dots,e_N^T(t))^T$, then the following error system is obtained:

$$\dot{\varepsilon}(t)=(-\bar{L}+\mathbf{1}_{N-1}\cdot\bar{l}_1^T)(\varepsilon(t-\tau)+e(t))-d\otimes e_1(t) \tag{6}$$

where $d = (l_{21}-l_{11},\dots,l_{N1}-l_{11})^T$. Note that when we discuss the consensus problem, $\|\varepsilon(t)\|$ becomes a convenient study object instead of $\|x(t)\|$ because $\|\varepsilon(t)\| = 0$ is equal to $\|x(t)\| = 0$ with $t \to \infty$.

Theorem 1. Consider agent system (2) with event-triggered control strategy (5). Assume the communication topology has a directed spanning tree. Then the consensus is reached asymptotically for any $\beta_1 \in \left(0, \frac{\zeta}{\sigma(1+\zeta)}\right), \beta_2 > 0, 0 < \gamma < \alpha$ when the time delay $\tau < \tau_0$, where σ, ζ, τ_0 are estimated in the following (7), (18), (19), and the parameter α can be determined by matrix $\tilde{L}$ according to Lemma 3.

Proof. We first have the following derivation

$$\begin{aligned}
&\sum_{j\in\mathcal{N}_i} \|x_j\left(t^j_{k_j} - \tau\right) - x_i\left(t^i_{k_i} - \tau\right)\| \\
&= \sum_{j\in\mathcal{N}_i} \|x_j(t-\tau) - x_1(t-\tau) - (x_i(t-\tau) \\
&\quad - x_1(t-\tau)) + e_j(t) - e_i(t)\| \\
&\le \sum_{j\in\mathcal{N}_i} \left(\|\varepsilon_j(t-\tau) - \varepsilon_i(t-\tau)\| + \|e_j(t) - e_i(t)\|\right) \\
&\le \sum_{j\in\mathcal{N}_i} \|\varepsilon_j(t-\tau)\| + N_i\|\varepsilon_i(t-\tau)\| + \sum_{j\in\mathcal{N}_i} \|e_j(t)\| \\
&\quad + N_i\|e_i(t)\| \\
&\le \left(\sqrt{N_i} + N_i\right)(\|\varepsilon(t-\tau)\| + \|e(t)\|).
\end{aligned}$$

Note that the above derivation depends on the inequality $\left(a_1 + \ldots + a_n\right)/n \le \sqrt{\left(a_1^2 + \ldots + a_n^2\right)/n}$. Then, from (5) one has

$$\begin{aligned}
\|e_i(t)\| &\le \frac{\beta_1}{N_i}\left(\sqrt{N_i} + N_i\right)(\|\varepsilon(t-\tau)\| + \|e(t)\|) + \beta_2 e^{-\gamma t} \\
&\le \beta_1\left(\frac{1}{\sqrt{\bar{N}}} + 1\right)(\|\varepsilon(t-\tau)\| + \|e(t)\|) + \beta_2 e^{-\gamma t},
\end{aligned}$$

where $\bar{N} = \min\{N_1, \ldots, N_N\}$. Thus, one can further obtain

$$\|e(t)\| \le \kappa_1\|\varepsilon(t-\tau)\| + \kappa_2 e^{-\gamma t}, \tag{7}$$

where $\kappa_1 = \frac{\sigma\beta_1}{1-\sigma\beta_1}, \kappa_2 = \frac{\beta_2\sqrt{N}}{1-\sigma\beta_1}, \sigma = \sqrt{N}\left(\frac{1}{\sqrt{\bar{N}}} + 1\right)$.

By the Leibnitz formula, we have $\varepsilon(t-\tau) = \varepsilon(t) - \int_{t-\tau}^{t} \dot{\varepsilon}(z)dz$ for all differentiable functions ε. Let $\tilde{L} = \bar{L} - \mathbf{1}_{N-1} \cdot \bar{l}_1^T$, then the system (6) can be rewritten as:

$$\begin{aligned}
\dot{\varepsilon}(t) &= -\tilde{L}\varepsilon(t) + \tilde{L}\int_{t-\tau}^{t} \dot{\varepsilon}(z)dz - \tilde{L}e(t) - d \otimes e_1(t) \\
&= -\tilde{L}\varepsilon(t) + \tilde{L}\int_{t-\tau}^{t} [-\tilde{L}(\varepsilon(z-\tau) + e(z)) \\
&\quad - d \otimes e_1(z)]dz - \tilde{L}e(t) - d \otimes e_1(t)
\end{aligned} \tag{8}$$

Then we can get the following solution:

$$\begin{aligned}
\varepsilon(t) = e^{-\tilde{L}t}\varepsilon(0) + \int_0^t e^{-\tilde{L}(t-s)}\Big\{\tilde{L}\int_{s-\tau}^{s} \big[-\tilde{L}(\varepsilon(z-\tau) \\
+ e(z)) - d \otimes e_1(z)\big]dz - \tilde{L}e(s) - d \otimes e_1(s)\Big\}ds
\end{aligned} \tag{9}$$

According to Lemma 2, matrix $\tilde{L} = \bar{L} - \mathbf{1}_{N-1} \cdot \bar{l}_1^T$ is positive stable, namely, all the eigenvalues of $-\tilde{L}$ have negative real parts. Then, from Lemma 3, we can realize there exist a constant $\rho \ge 1$ and $\alpha > 0$ such that $\|e^{-\tilde{L}t}\| \le \rho e^{-\alpha t}$; thus,

$$\begin{aligned}
\|\varepsilon(t)\| &\le \rho\|\varepsilon(0)\|e^{-\alpha t} + \rho\int_0^t e^{-\alpha(t-s)}\Big\{\int_{s-\tau}^{s} \|\tilde{L}\|\big[\|\tilde{L}\| \\
&\quad \cdot\|\varepsilon(z-\tau)\| + (\|\tilde{L}\| + \|d\|)\|e(z)\|\big]dz \\
&\quad + (\|\tilde{L}\| + \|d\|)\|e(s)\|\Big\}ds \\
&\le \rho\|\varepsilon(0)\|e^{-\alpha t} + \rho\int_0^t e^{-\alpha(t-s)}\Big\{\int_{s-\tau}^{s} \|\tilde{L}\|\big[\|\tilde{L}\| \\
&\quad \cdot\|\varepsilon(z-\tau)\| + (\|\tilde{L}\| + \|d\|)\left(\kappa_1\|\varepsilon(z-\tau)\|\right. \\
&\quad \left.+ \kappa_2 e^{-\gamma z}\right)\big]dz + (\|\tilde{L}\| + \|d\|)\left(\kappa_1\|\varepsilon(s-\tau)\|\right. \\
&\quad \left.+ \kappa_2 e^{-\gamma s}\right)\Big\}ds \\
&= \rho\|\varepsilon(0)\|e^{-\alpha t} + \rho\int_0^t e^{-\alpha(t-s)}\Big\{\int_{s-\tau}^{s} \|\tilde{L}\|\big[\|\tilde{L}\| \\
&\quad \cdot\|\varepsilon(z-\tau)\| + \kappa_1(\|\tilde{L}\| + \|d\|)\|\varepsilon(z-\tau)\|\big]dz \\
&\quad + \kappa_1(\|\tilde{L}\| + \|d\|)\|\varepsilon(s-\tau)\|\Big\}ds + \rho\chi(e^{-\gamma t} \\
&\quad - e^{-\alpha t}).
\end{aligned} \tag{10}$$

where $\chi = \frac{\kappa_2(\|\tilde{L}\|+\|d\|)(\|\tilde{L}\|(e^{\gamma\tau}-1)+\gamma)}{\gamma(\alpha-\gamma)}$.

In the following, we will prove that

$$\|\varepsilon(t)\| \le \rho\varphi e^{-\gamma t}, \quad \gamma < \alpha, t \ge 0, \tag{11}$$

where $\varphi = \max\left\{\|\varepsilon(0)\|, \frac{\gamma(\alpha-\gamma)\chi}{\gamma(\alpha-\gamma)-\rho\varpi}\right\}$ and ϖ is defined in (14).

One can see that, Inequality (11) is equal to the following:

$$\|\varepsilon(t)\| < \eta\rho\varphi e^{-\gamma t}, \quad \eta > 1, t \geq 0. \tag{12}$$

Using the proof by contradiction, we assume Inequality (12) is impossible. Then due to the continuity of $\|\varepsilon(t)\|$, there must exist a time instant $t^* > 0$ such that

$$\|\varepsilon(t^*)\| = \eta\rho\varphi e^{-\gamma t^*} = \psi(t^*). \tag{13}$$

However, from (10) and (12), we obtain that

$$\begin{aligned}
\|\varepsilon(t^*)\| &\leq \rho\|\varepsilon(0)\|e^{-\alpha t^*} + \rho\chi\left(e^{-\gamma t^*} - e^{-\alpha t^*}\right) \\
&\quad + \rho\int_0^{t^*} e^{-\alpha(t^*-s)}\left\{\int_{s-\tau}^{s}\left[\left(\|\tilde{L}\|^2 + \kappa_1\|\tilde{L}\|\right.\right.\right. \\
&\quad \left.\cdot(\|\tilde{L}\| + \|d\|)\right)\|\varepsilon(z-\tau)\|\Big] dz + \kappa_1\left(\|\tilde{L}\|\right. \\
&\quad \left.+\|d\|\right)\|\varepsilon(s-\tau)\|\Big\} ds \\
&< \eta\rho\|\varepsilon(0)\|e^{-\alpha t^*} + \eta\rho\chi\left(e^{-\gamma t^*} - e^{-\alpha t^*}\right) \\
&\quad + \rho\int_0^{t^*} e^{-\alpha(t^*-s)}\left\{\int_{s-\tau}^{s}\left[\left(\|\tilde{L}\|^2 + \kappa_1\|\tilde{L}\|\right.\right.\right. \\
&\quad \left.\cdot(\|\tilde{L}\| + \|d\|)\right)\eta\rho\varphi e^{-\gamma(z-\tau)}\Big] dz + \kappa_1\left(\|\tilde{L}\|\right. \\
&\quad \left.+\|d\|\right)\eta\rho\varphi e^{-\gamma(s-\tau)}\Big\} ds \\
&= \eta\rho\|\varepsilon(0)\|e^{-\alpha t^*} + \eta\rho\left(\chi + \frac{\rho\varphi\varpi}{\gamma(\alpha-\gamma)}\right)(e^{-\gamma t^*} \\
&\quad - e^{-\alpha t^*}),
\end{aligned} \tag{14}$$

where $\varpi = \left[\|\tilde{L}\|^2 + \kappa_1\|\tilde{L}\|(\|\tilde{L}\| + \|d\|)\right](e^{2\gamma\tau} - e^{\gamma\tau}) + \gamma\kappa_1(\|\tilde{L}\| + \|d\|)e^{\gamma\tau}$.

Without loss of generality, we analyze two cases here to show Inequality (12) always holds.

Case 1. $\varphi = \|\varepsilon(0)\|$, *i.e.*, $\|\varepsilon(0)\| - \left(\chi + \frac{\rho\varphi\varpi}{\gamma(\alpha-\gamma)}\right) > 0$. We have the following derivation:

$$\begin{aligned}
\|\varepsilon(t^*)\| &< \eta\rho\|\varepsilon(0)\|e^{-\alpha t^*} + \eta\rho\|\varepsilon(0)\|\left(e^{-\gamma t^*} - e^{-\alpha t^*}\right) \\
&= \eta\rho\|\varepsilon(0)\|e^{-\gamma t^*} \\
&= \eta\rho\varphi e^{-\gamma t^*} = \psi(t^*).
\end{aligned} \tag{15}$$

Case 2. $\varphi = \frac{\gamma(\alpha-\gamma)\chi}{\gamma(\alpha-\gamma)-\rho\varpi}$, *i.e.*, $\|\varepsilon(0)\| - \left(\chi + \frac{\rho\varphi\varpi}{\gamma(\alpha-\gamma)}\right) < 0$. We have another derivation:

$$\begin{aligned}
\|\varepsilon(t^*)\| &< \eta\rho\left(\chi + \frac{\rho\varphi\varpi}{\gamma(\alpha-\gamma)}\right)\left(e^{-\alpha t^*} + e^{-\gamma t^*} - e^{-\alpha t^*}\right) \\
&= \eta\rho\frac{\gamma(\alpha-\gamma)\chi}{\gamma(\alpha-\gamma)-\rho\varpi}e^{-\gamma t^*} \\
&= \eta\rho\varphi e^{-\gamma t^*} = \psi(t^*).
\end{aligned} \tag{16}$$

From derivation (15) and (16), we see that $\|\varepsilon(t^*)\| < \psi(t^*)$, which is contradictory with (13). Thus, Inequalities (12) and (11) hold, which implies the consensus is reached exponentially with $t \to \infty$.

Next, we begin to discuss the existence of the delay upper bound. From (11) and (14), we can obtain

$$\chi + \frac{\rho\varphi\varpi}{\gamma(\alpha-\gamma)}) < \varphi.$$

One can see that the inequality $\frac{\rho\varphi\varpi}{\gamma(\alpha-\gamma)} < \varphi$ is satisfied. Let

$$\begin{aligned}
g(\tau) &= \rho\varpi - \gamma(\alpha-\gamma) \\
&= \rho\delta_1 e^{2\gamma\tau} + \rho(\delta_2 - \delta_1)e^{\gamma\tau} + \gamma^2 - \alpha\gamma,
\end{aligned}$$

where $\delta_1 = \|\tilde{L}\|^2 + \kappa_1\|\tilde{L}\|(\|\tilde{L}\| + \|d\|)$, $\delta_2 = \gamma\kappa_1(\|\tilde{L}\| + \|d\|)$. We have

$$g'(\tau) = \rho\delta_1\gamma(2e^{2\gamma\tau} - e^{\gamma\tau}) + \rho\delta_2\gamma e^{\gamma\tau} > 0.$$

Further, when the following is satisfied

$$\begin{cases} g(0) = \rho\delta_2 + \gamma^2 - \alpha\gamma < 0 \\ g(\infty) = \infty, \end{cases} \tag{17}$$

there exist unique τ_0 such that $g(\tau) < 0$ when $\tau \in [0, \tau_0)$. Then, to guarantee (17) is satisfied, we can solve

$$\beta_1 < \frac{\zeta}{\sigma(\zeta+1)} \tag{18}$$

where $\zeta = \frac{\alpha-\gamma}{\rho(\|\tilde{L}\|+\|d\|)}$, and letting $g(\tau_0) = 0$, the following delay upper bound is obtained

$$\tau_0 = \frac{1}{\gamma}\ln\frac{\delta_1 - \delta_2 + \sqrt{(\delta_2-\delta_1)^2 - 4\delta_1(\gamma^2/\rho - \alpha\gamma/\rho)}}{2\delta_1} \tag{19}$$

Remark 2. Here, we should give the interpretation of the global objective consensus state of an agent network containing a directed spanning tree. Yu and Chen investigated the consensus problem of a directed agent network in [28], from which we can regard a directed spanning tree as a composition of several strongly connected subgraphs. Then, we can analyze the consensus problem using the existing methods from strongly connected network topology. According to [28], the global network objective consensus state is only determined by the first strongly connected subgraph whose ultimate consensus state is the initial weighted states average of its agents.

Definition 2. [29]. If there exists a finite C such that $t_{k+1} - t_k = 0$ for all $k > C$, then the Zeno behavior take place.

Generally speaking, Zeno behavior is a phenomenon where there are infinitely many discrete transitions occurring in a finite time interval. The realistic physical systems are, of course, not Zeno, but the hybrid model of a physical system may be Zeno, due to modeling over-abstraction. Once the Zeno behavior happens, the computer simulations become imprecise and time-consuming. Thus, the Zeno behavior must be excluded in model analysis.

Corollary 1. Consider the agent system (4) with the event-triggered control strategy (5). Assume the communication topology has a directed spanning tree. Then, the inter-event times for each agent $i = 1, 2, \dots, N$ are strictly positive.

Proof. We first let

$$z_i\left(t^i_{k_i}, t^j_{k_j}\right) = \sum_{j=1}^{N} a_{ij}\left(x_i\left(t^i_{k_i} - 2\tau\right) - x_j\left(t^j_{k_j} - 2\tau\right)\right), \tag{20}$$

then we take the derivative of $\|e_i(t)\|$ at time interval $t \in \left[t^i_{k_i}, t^i_{k_i+1}\right)$ when $\|e_i(t)\|$ is continuous:

$$\begin{aligned} &\tfrac{d}{dt}\|e_i(t)\| \le \|\dot{e}_i(t)\| = \|\dot{x}_i(t-\tau)\| \\ &= \|z_i(t^i_{k_i}, t^j_{k_j})\|. \end{aligned} \tag{21}$$

Consider the differential equation at time interval $t \in [t^i_{k_i}, t^i_{k_i+1})$

$$\begin{cases} \dot{\phi}_i(t) = \|z_i\left(t^i_{k_i}, t^j_{k_j}\right)\| \\ \phi_i\left(t^i_{k_i}\right) = \|e_i\left(t^i_{k_i}\right)\| = 0, \end{cases} \tag{22}$$

then we have

$$\|e_i(t)\| \le \phi_i(t) = \int_{t^i_{k_i}}^{t} \|z_i\left(t^i_{k_i}, t^j_{k_j}\right)\| d\tau. \tag{23}$$

The next event triggering time of agent i is obtained by finding the minimum time t such that $\phi_i(t) > \frac{\beta_1}{N_i}\sum_{j\in\mathcal{N}_i}\|x_j\left(t^j_{k_j}\right) - x_i\left(t^i_{k_i}\right)\| + \beta_2 e^{-\gamma(t)} > 0$ before the consensus is reached. Note that we have to analyze two cases here, the first case is when $\|z_i\left(t^i_{k_i}, t^j_{k_j}\right)\| \neq 0$ at the last update instant $t^i_{k_i}$. Since $\dot{\phi}_i(t) > 0$ and $\phi_i\left(t^i_{k_i}\right) = 0$, the minimum time t corresponding to $\phi_i(t) > 0$ must be larger than $t^i_{k_i}$. The other case is when $\|z_i\left(t^i_{k_i}, t^j_{k_j}\right)\| = 0$ during $t \in \left[t^i_{k_i}, t^j_{k_j}\right)$, which only happens occasionally. In this case, $\phi_i(t) = 0$ and we see Inequality (5) holds, then the agent i does not generate any events during that time interval. When agent i receives an update from its neighbors, then $\|z_i\left(t^i_{k_i}, t^j_{k_j}\right)\| \neq 0$ and the first case holds. Therefore, the inter-event times for each agent $i = 1, 2, \dots, N$ are strictly positive, which implies agent i will not exhibit Zeno triggering behavior. The proof is complete.

IV. SIMULATIONS

In this section, we provide some simulations to illustrate the proposed approach. Consider the information interactive network with communication graph $\mathcal{G}$ given in Fig. 1, and consider the initial state values of agents are randomly generated in the interval $[-5, 5]$.

According to Lemma 3 and the topological structure in Fig. 1, we obtain $\rho = 4.5783, \alpha = 0.3820$. Then, we set $\beta_2 = 0.2, \gamma = 0.01$ and calculate $\beta_1 < 0.0018$. Letting $\beta_1 = 0.0012$, the time delay bound $\tau_0 = 0.0016$ is calculated, then a group of simulation results are obtained.

Fig. 2 shows the evolution of all agents states $x_i(t)$. Fig. 3 shows the piecewise constant control signals $u_{i1}(t)$. The evolution of measurement error norm of the first agent is shown in Fig. 4, from which we cannot directly

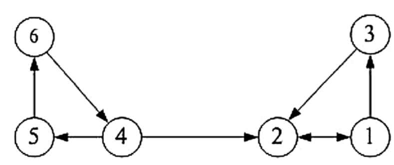


Fig. 1. The interaction diagraph with 6 agents.

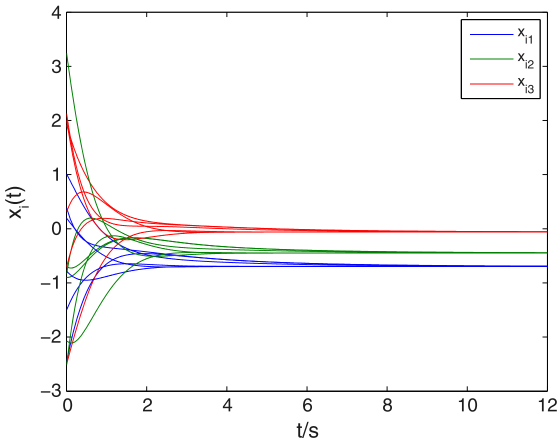


Fig. 2. The evolution of states $x_i(t)$, $i = 1, 2, \dots, 6$.

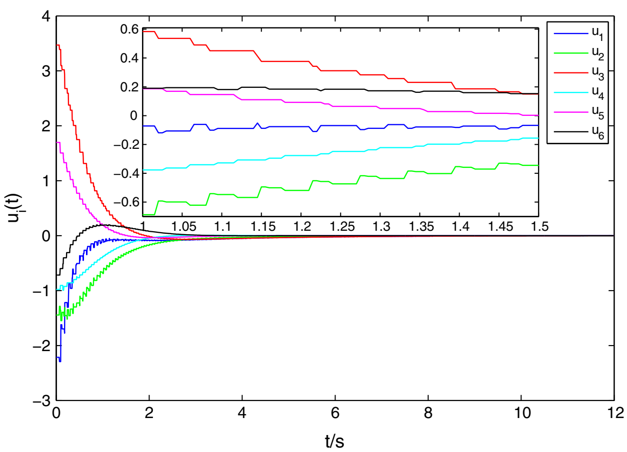


Fig. 3. The evolution of control signals $u_{i1}(t)$, $i = 1, 2, \dots, 6$.

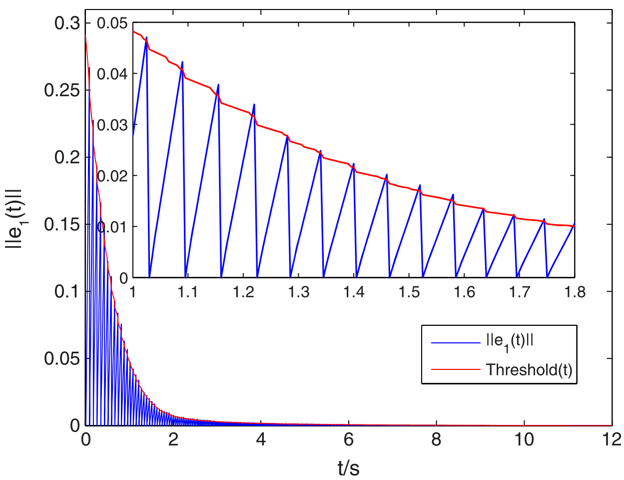


Fig. 4. The evolution of measurement error and threshold.

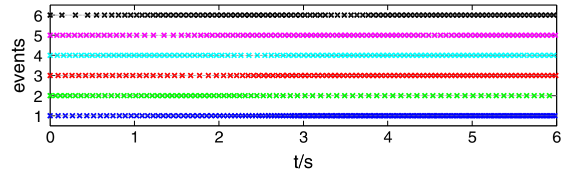


Fig. 5. Event triggering times of 6 agents.

see the threshold is a piecewise constant function because the function $e^{-\gamma t}$ is continuous. In Fig. 5, the events of each agent are marked in time interval $[0, 6]$, from which we can see that the sampling is sporadic rather than at every time instant.

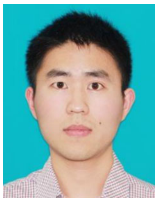
V. CONCLUSION

In this paper, we have proposed a novel event triggering function that not only truly avoids continuous communication but also excludes the Zeno behavior for event-triggered control in multi-agent systems with communication delays. Then, we provide the consensus analysis using an inequality technique instead of the traditional LMI method and demonstrate that the Zeno behavior can be excluded. However, the event triggering mechanism still has many new challenges to overcome in real and more complicated conditions. Future works will include extending the proposed approach to interactive systems with second-order dynamics.

REFERENCES

1. DeGroot, M. H., "Reaching a consensus," *J. Am. Statist. Assoc.*, Vol. 69, No. 345, pp. 118–121 (1974).
2. Li, H., X. Liao, X. Lei, T. Huang, and W. Zhu, "Second-order consensus seeking in multi-agent systems with nonlinear dynamics over random switching directed networks," *IEEE Trans. Circuits Syst. I-Regul. Pap.*, Vol. 60, No. 6, pp. 1595–1607 (2013).
3. Li, H., X. Liao, and T. Huang, "Second-order dynamic consensus of multi-agent systems with arbitrarily fast switching topologies," *IEEE Trans. Syst. Man Cybern. Part A-Syst. Man*, Vol. 43, No. 6, pp. 1343–1353 (2013).
4. Li, H., X. Liao, T. Huang, W. Zhu, and Y. Liu, "Second-order globally nonlinear consensus in multi-agent networks with random directional link failure," *IEEE Trans. Neural Netw. Learn. Syst.*, Vol. 26, pp. 565–575 (2015).

5. Fax, J. A. and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control*, Vol. 49, No. 9, pp. 1465–1476 (2004).
6. Zhang, H.-T., Z. Chen, L. Yan, and W. Yu, "Applications of collective circular motion control to multi-robot systems," *IEEE Trans. Control Syst. Technol.*, Vol. 21, No. 4, pp. 1416–1422 (2013).
7. Lin, J., A. S. Morse, and B. D. O. Anderson, "The multi-agent rendezvous problem–part 1 the synchronous case," *SIAM J. Control Optim.*, Vol. 46, pp. 2096–2119 (2007).
8. Lin, J., A. S. Morse, and B. D. O. Anderson, "The multi-agent rendezvous problem–part 2 the asynchronous case," *SIAM J. Control Optim.*, Vol. 46, pp. 2120–2147 (2007).
9. Tanner, H. G., A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. Autom. Control*, Vol. 52, pp. 863–868 (2007).
10. Olfati-Saber, R. and J. S. Shamma, "Consensus filters for sensor networks and distributed sensor fusion," *44th IEEE Conf. Decis. Control and Eur. Control Conf. (CDC-ECC'05)*, Seville, Spain, pp. 6698–6703 (2005).
11. Albert, A., "Comparison of event-triggered and time-triggered concepts with regard to distributed control systems," *Embed. World*, Vol. 2004, pp. 235–252 (2004).
12. Tabuada, P., "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Autom. Control*, Vol. 52, pp. 1680–1685 (2007).
13. Wang, X. and M. D. Lemmon, "Event-triggered broadcasting across distributed networked control systems," *Proc. Amer. Control Conf.*, Seatle, WA, USA, pp. 3139–3144 (2008).
14. Dimarogonas, D. V. and K. H. Johansson, "Event-triggered control for multi-agent systems," *Proc. 48th IEEE Conf. Decis. Control and 28th Chinese Control Conf. (CDC/CCC)*, Shanghai, China, pp. 7131–7136 (2009).
15. Dimarogonas, D. V., E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Trans. Autom. Control*, Vol. 57, pp. 1291–1297 (2012).
16. Garcia, E., Y. Cao, H. Yu, P. Antsaklis, and D. Casbeer, "Decentralised event-triggered cooperative control with limited communication," *Int. J. Control*, Vol. 86, pp. 1479–1488 (2013).
17. Nowzari, C. and J. Cortés, "Zeno-free, distributed eventtriggered communication and control for multi-agent average consensus," *in Amer. Control Conf.*, Portland, OR, pp. 2148–2153 (2014).
18. Li, H., X. Liao, T. Huang, and W. Zhu, "Event-triggering sampling based leader-following consensus in second-order multi-agent systems," *IEEE Trans. Autom. Control* (2015).
19. Gao, L., X. Liao, and H. Li, "Pinning controllability analysis of complex networks with a distributed event-triggered mechanism," *IEEE Trans. Circuits Syst. II-Brief Papers*, Vol. 61, No. 7, pp. 541–545 (2014).
20. Fan, Y., G. Feng, Y. Wang, and C. Song, "Distributed event-triggered control of multi-agent systems with combinational measurements," *Automatica*, Vol. 49, pp. 671–675 (2013).
21. Zhu, W., Z. Jiang, and G. Feng, "Event-based consensus of multi-agent systems with general linear models," *Automatica*, Vol. 50, pp. 552–558 (2014).
22. Zhu, W. and Z. Jiang, "Event-based leader-following consensus of multi-agent systems with input time delay," *IEEE Trans. Autom. Control*, Vol. 60, pp. 1362–1367 (2015). DOI: 10.1109/TAC.2014.2357131.
23. Zhang, H., Y. Ren, and X. Wang, "Distributed event-triggered quantizer in multi-agent systems," *J. Dynam. Syst. Meas. Control-Trans. ASME, Vol. 136, No. 4, pp. 044504* (2014).
24. Olfati-Saber, R. and R. M. Murray, "Consensus problems networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, Vol. 49, pp. 1520–1533 (2004).
25. Pepe, P. and Z. P. Jiang, "A Lyapunov–Krasovskii methodology for iss and iiss of time-delay systems," *Syst. Control Lett.*, Vol. 55, pp. 1006–1014 (2006).
26. Zhu, W. and D. Cheng, "Leader-following consensus of second-order agents with multiple time-varying delays," *Automatica*, Vol. 46, pp. 1994–1999 (2010).
27. Wu, C. W., *Synchronization in Complex Networks of Nonlinear Dynamical Systems*, World Scientific Press, Singapore (2007).
28. Yu, W., G. Chen, and M. Cao, "Consensus in directed networks of agents with nonlinear dynamics," *IEEE Trans. Autom. Control*, Vol. 56, No. 6, pp. 1436–1441 (2011).
29. Ames, A. D., A. Abate, and S. Sastry, "Sufficient conditions for the existence of Zeno behavior," *44th IEEE Conf. Decis. Control and 2005 Eur. Control Conf. (CDC-ECC'05)*, Seville, Spain, pp. 696–701 (2005).



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